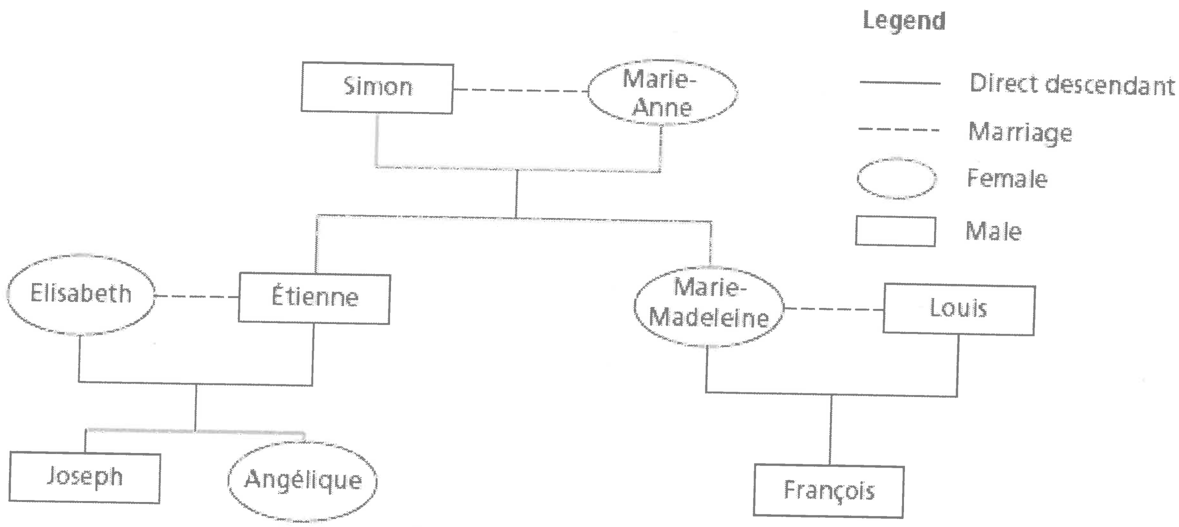
Notes Ch. 5

5.1 Representing Relations p. 256

This family tree shows relations within a family.



How is Joseph related to Simon? grandson

How are Angélique and François related? cousin

How does the family tree show these relations?
- lines → different kinds
- some names under others
- different shapes

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- a **set** is: a collection of distinct objects (in our case, numbers)

- An **element** ~~is~~: of a set; is one distinct object (number) in the set.

One way to write a set is to list its elements in brackets.

*For example we can write the set of real numbers from 1 to 5 as: **R** = {1,2,3,4,5}

*Any of the numbers in the braces is an element of that set.

(1,2,3,4,5 are each **elements** of the **set R** of real numbers from 1 to 5.)

The order of the elements in the set does not matter.

A relation associates the ___elements___ of one set with the ___elements___ of another set.

- A relation is a rule that **associates** the elements of one set with the elements of another set (like a set of ordered pairs).
- A relation produces **one or more output** numbers for every valid input number.

When we represent **relations** using numbers, a relation is a set of $\boxed{\text{ordered pairs.}}$ The elements in the relation are the numbers that represent specific coordinate points on a Cartesian plane.

ie.
{(2,4), (4,8), (6,12)} is a relation
{(boy, 11)} is a relation
{(8:37, bell)} is a relation
{(beef, $17)} is a relation

In this example of a relation (*a set of ordered pairs*): {(1, a), (2, b), (3, c) }, the set of first elements is called the **domain** { 1, 2, 3 }and the set of second elements is called the **range** {a, b, c}.

Ways to represent relations:

Relations can be represented in a number of ways:
1. Table of Values
2. Graphs
3. Arrow Diagrams (mapping diagram)
4. Equations
5. In Words (description)

*

Example 1 – Represent the following relation in 4 different ways:

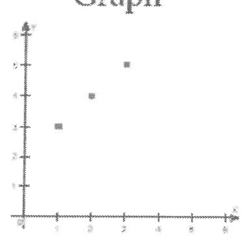
{(1,3), (2,4), (3,5)}

Recall: (*x*, *y*)

Table of Values

x	y
1	3
2	4
3	5

Graph



Equation

$y = x + 2$

Words

y is always equal to the value of *x* plus two.

Example 2 - represent a relation in words, in table, in arrow diagram

➔**Consider the set of fruits and the set of colors.**

- We can associate fruits with their colors in words.

An apple	may have the colour	red.
↑	↑	↑
element of first set	association	element of second set

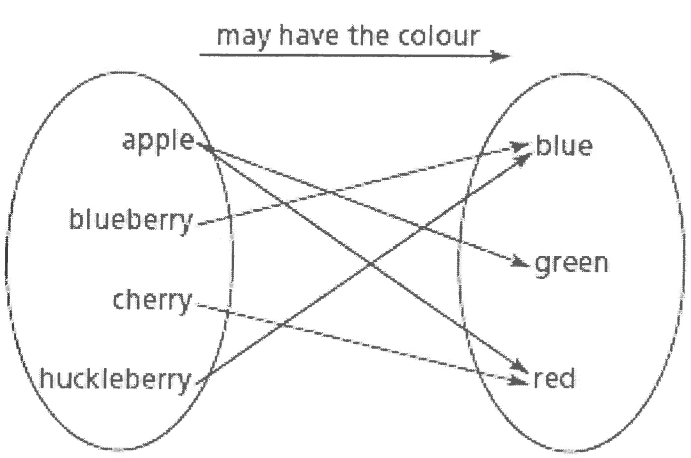
- So the set of **ordered pairs** is a relation.

{(apple, red), (apple, green), (blueberry, blue), (cherry, red), (huckleberry, blue)}

- We can represent the relation in a **table**. The heading of each column describes each set.

Fruit	Colour
apple	red
apple	green
blueberry	blue
cherry	red
huckleberry	blue

- We can represent the relation in an **arrow diagram** *(mapping diagram)*. Two **ovals** describe each **set.**
  Each **arrow** associates an element in the 1st set to the 2nd set.



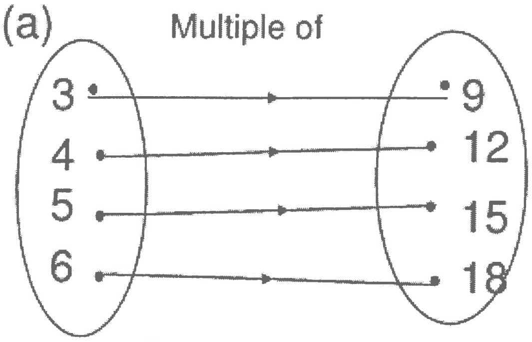
The order of the words in the ordered pairs and which column and which oval the words are in in the table and arrow diagram are important , as is the direction of the arrows.

It makes sense to say "an apple may be the color red" but does not make sense to say "red may be the color apple".

That is, **a relation has direction from one set to the other set.**

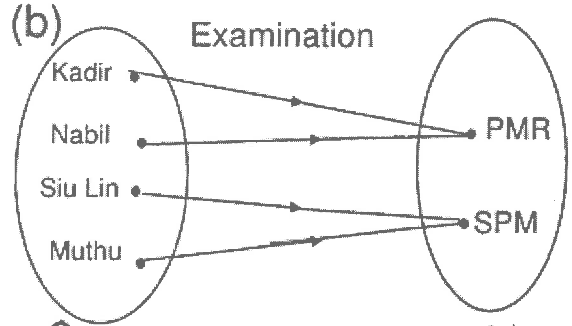
The four main types of relations are shown in the following arrow diagrams.

(a)



One to one

(b)

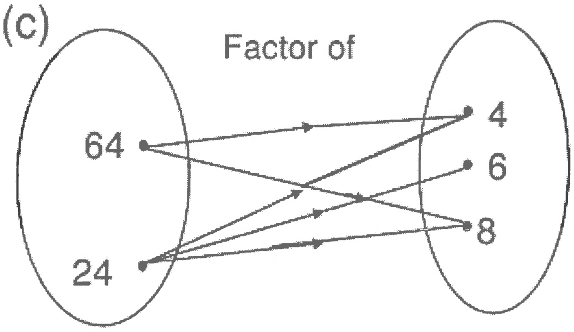


names

many to one

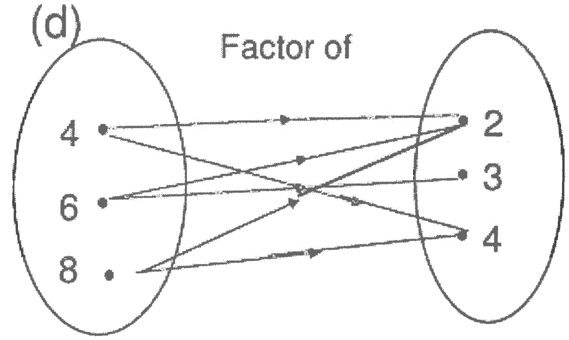
typed names written

(c)



One to many

(d)



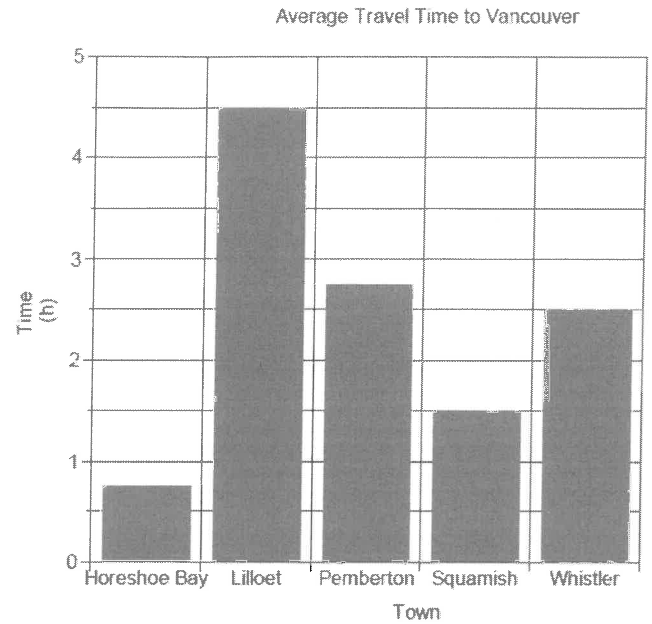
Many to many

Sometimes a relation contains so many ordered pairs that it is impossible to list all of them or to represent them in a table or an arrow diagram.

Representing Relations in words, as a graph, as a table, as an arrow diagram.

Ex. Different towns in British Columbia can be associated with the average time. in hours it takes to drive to Vancouver. Consider the relation represented by the following graph.

← words

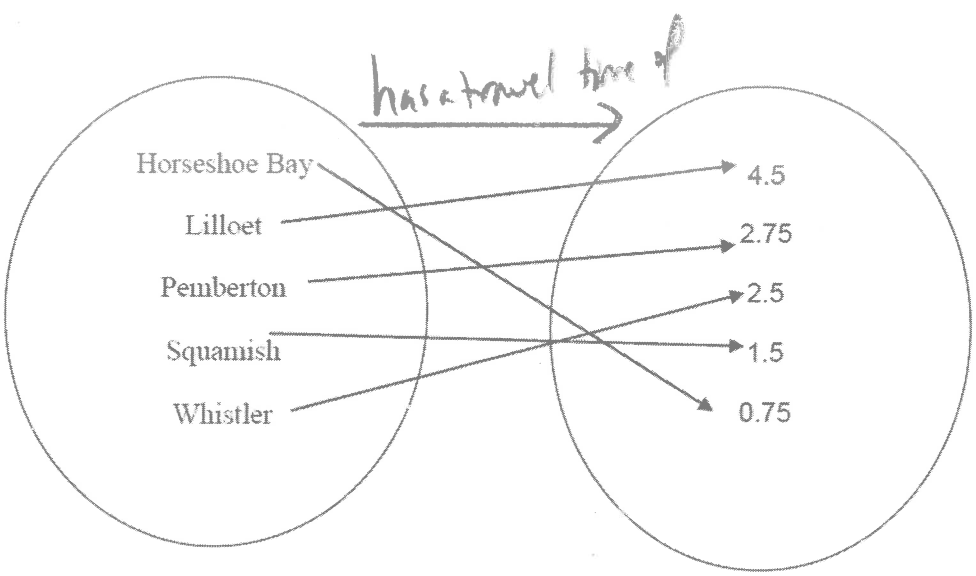


This graph shows the relation "travel time to Vancouver" between a set of towns and a set of travel times.

a) As a table

Town	Average Time(h)
Horseshoe Bay	0.75
Lilloet	4.5
Pemberton	2.75
Squamish	1.5
Whistler	2.5

b) as an arrow diagram:



c) As a set of ordered pairs:

{(Horseshoe Bay, 0.75), (Lilloet, 4.5), (Pemberton, 2.75), (Squamish, 1.5), (Whistler, 2.5)}

Try these:

Animal	Class
Ant	Insecta
Eagle	Aves
Snake	Reptilia
Turtle	Reptilia
Whale	Mammalia

1. DESCRIBE THE ABOVE RELATION IN WORDS.

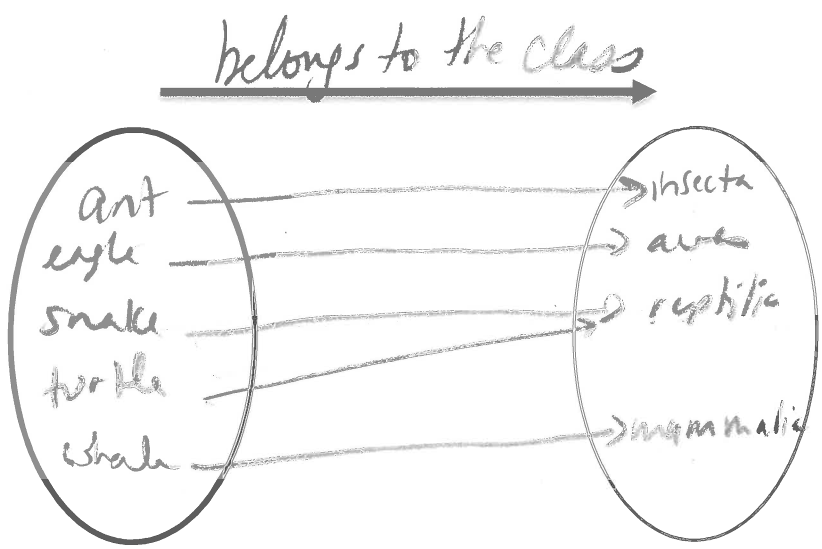
The relation shows the association "belongs to the class" between a set of animals and a set of classes.

Ant $\xrightarrow{\text{belongs to the class}}$ Insecta

2. REPRESENT AS A SET OF ORDERED PAIRS

{(ant, insecta); (eagle, Aves); (snake, reptilia); (turtle, reptilia); (whale, mammal)}

3. REPRESENT AS AN ARROW DIAGRAM.



When the elements of either one or both sets in a relation are in fact **numbers**, the relation can be represented as a bar graph as well (not always line graphs).

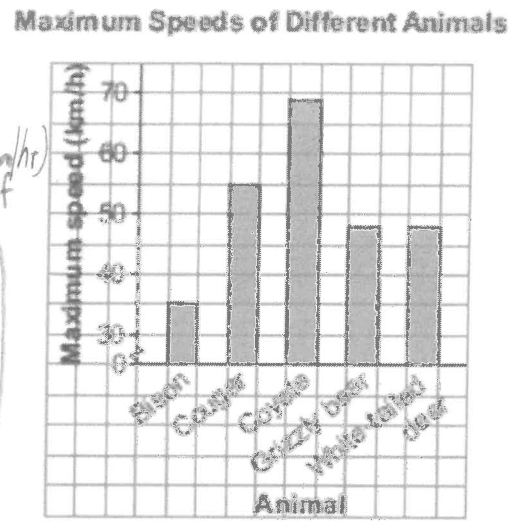
Words: The relation shows the association of "has a maximum speed" between a set of animals and a set of maximum speeds in km/h. For example, bison has a maximum speed of 35 km/h.

2.

Consider the relation represented by this graph. Represent the relation:

a) as a table

b) as an arrow diagram



animal	max speed (km/h)
bison	35
cougar	55
coyote	69
grizzly bear	48
white-tailed deer	48

has a max speed (km/hr) of

bison → 35
cougar → 55
coyote → 69
grizzly bear → 48
white-tailed deer → 48

Both sets in a relation can be the same.

Sometimes a relation contains so many ordered pairs that it is impossible to list all of them or to represent them in a table or an arrow diagram.

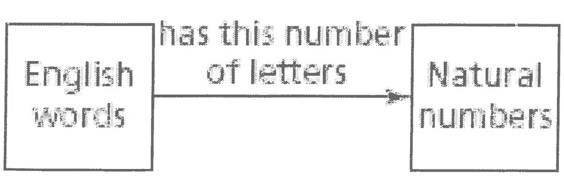
3. In the diagram below:

a) Describe the relation in words.

The relation shows the association "has this number of letters" for the set "English words" and "natural numbers".

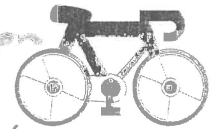
b) List 2 ordered pairs that belong to the relation.

(dog, 3) (bicycle, 7)



Introduction

Caitlin rides her bike to school every day. The table of values below shows her distance from home as time passes.



time (minutes) t	distance (metres) d
0	0
1	250
2	500
3	750
4	1000
5	1250

250

a) Write a sentence that describes this relation.

The relation shows the association "has a distance of" for the set "time (min)" and "distance from home (m)".

b) Represent this relation with ordered pairs.

{(0,0); (1,250); (2,500); (3,750); (4,1000); (5,1250)}

c) Represent this relation with an arrow diagram.

has a distance of (m)

- 0 → 0
- 1 → 250
- 2 → 500
- 3 → 750
- 4 → 1000
- 5 → 1250

d) Write an equation for this scenario.

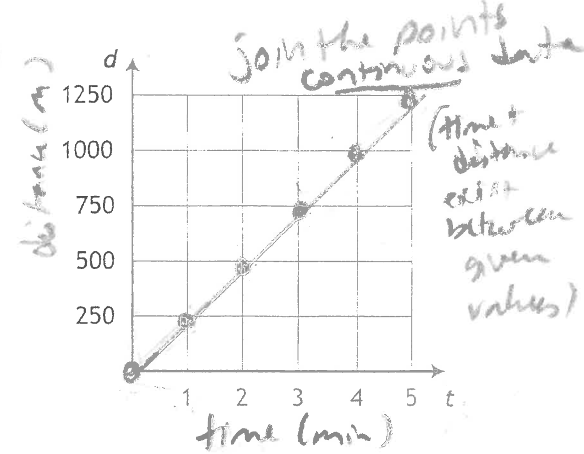
$d = 250t$

$d = 250(0) = 0$

$d = 250(1) = 250$

e) Graph the relation.

e)



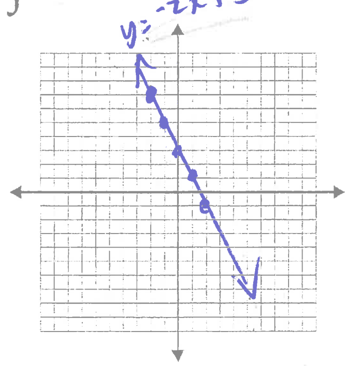
Example 1

For each relation, complete the table of values and draw the graph.

a) $y = -2x + 3$

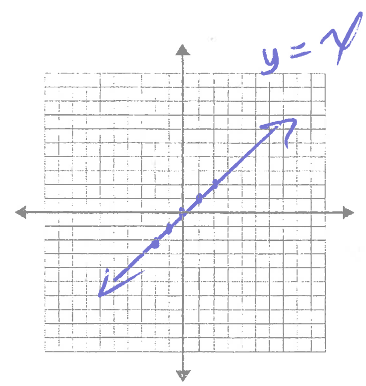
$y = -2(-2) + 3 = 7$

x	y
-2	7
-1	5
0	3
1	1
2	-1



b) $y = x$

x	y
-2	-2
-1	-1
0	0
1	1
2	2



a) + b)

Do page 262 #9, 12, 13, 14

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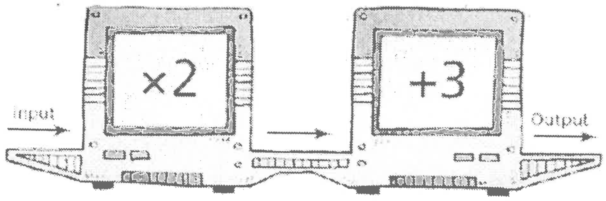


input

x Input	y Output
1	5
2	7
3	9
4	11
5	13

+1 +1 +1 +1)+2)+2)+2)+2

$y = 2x+3$
= output

What is the rule you see for this Input/Output machine above?

Multiply input number (first column of table) by 2. Then add 3 to the result. This rule produces the output (number in second column of table).

Which numbers would complete this table for the machine?

5.2 Properties of Functions - RECOGNITION OF FUNCTIONS VS RELATION

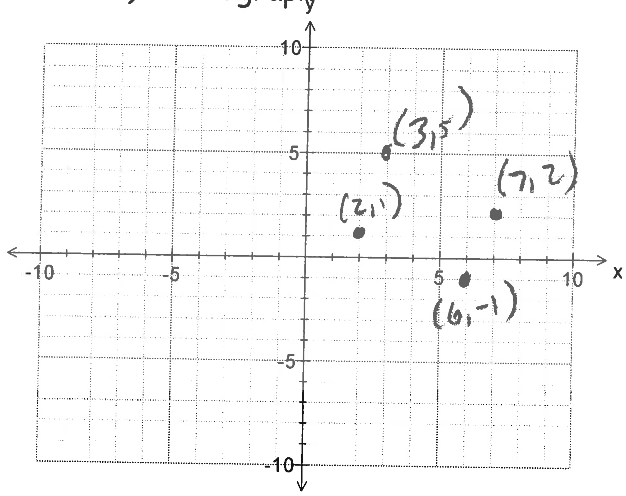
p. 265

- A relation is any set of ordered pairs. It represents a relationship between two quantities or a correspondence between two variables.

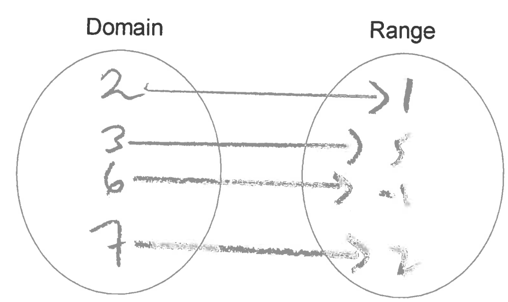
- Relations may be represented in many ways. The most common being ordered pairs, a table of values, a graph or a mapping diagram.

Example: The relation written as a list { (2, 1), (3, 5), (6, -1), (7, 2)} could be represented as
x, y

i) a graph



ii) a mapping diagram



iii) Table of Values

x	2	3	6	7	
y	1	5	-1	2	

The set of first elements (those typically corresponding to the x-values, or the independent variable) is the **DOMAIN**.

The set of second elements (those typically corresponding the y-values or the dependent variable) is called the **RANGE**.

A **FUNCTION** is a special type of relation where each element in the **domain** is used only once.

* A function is a type of relation for which each x value in the domain corresponds with only one y value.

Ex: { (2, 1), (3, 5), (6, -1), (7, 2)} is a function because there is only one y for each x.

Ex: {(2, 3), (3, 1), (5, 4), (2, 6)} is not a function because when x =2, there are 2 y-values.

Examples: **Are these functions?**

1) {(3, 8), (4, 9), (5, 10), (6, 11)}

yes

2) {(-1, 7), (-1, 5), (-1, 3), (-1, 1)}

no

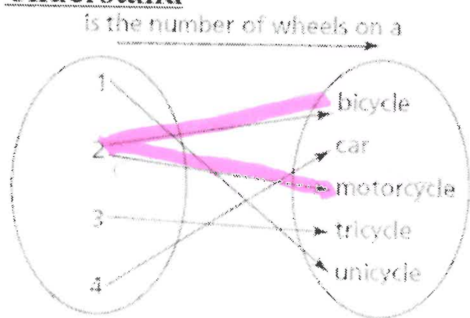
3) {(5, 9)}

yes

4) {(3, 3), (4, 4), (4, 5), (5, 5), (5, 6)}

no.

Understand



This relation associates a number with a vehicle with that number of wheels.

This diagram DOES NOT represent a function because there is one element in the first set that associates with TWO ELEMENTS in the second set.

One specific input value has more than one output value.

Example 1: For each relation below,

- Identify its domain and range.
- Decide whether the relation is a function.

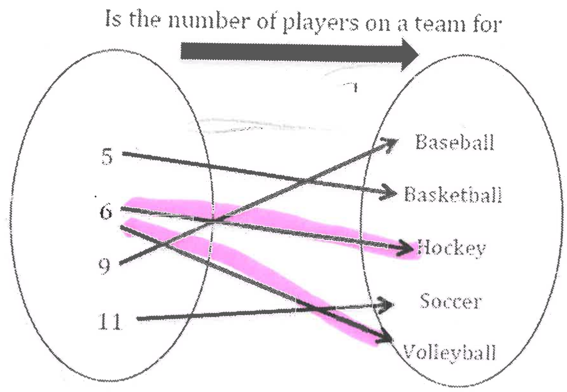
a) A relation that associates 5 foods to the food groups to which they belong: {(orange, fruit), (cheese, dairy), (broccoli, vegetables), (milk, dairy), (kiwi, fruit)}

Domain: { orange, cheese, broccoli, milk, kiwi }

Range: { fruit, dairy, vegetables }

Function? Yes each element of food goes to one food group. There isn't one food that goes to 2 food groups. Each element in first set associate with only one element in second set. Every ordered pair has a different first element. (It is okay to have food groups, second elements, repeat.)

b)



Domain: { 5, 6, 9, 11 }

Range: { baseball, basketball, hockey, soccer, volleyball }

Function? NO
6players associates with 2 sports.
1st element 2nd element

Write the relation in words:
The relation shows the association "is the number of players on a team" with the set of number of players and the set of sports.

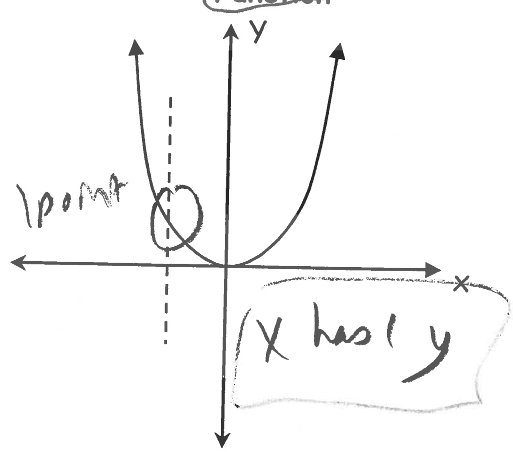
Write this relation as ordered pairs.
{ (5, basketball), (6, hockey), (6, volleyball), (9, baseball), (11, soccer) }

Straight line test to see whether or not a graph is a function

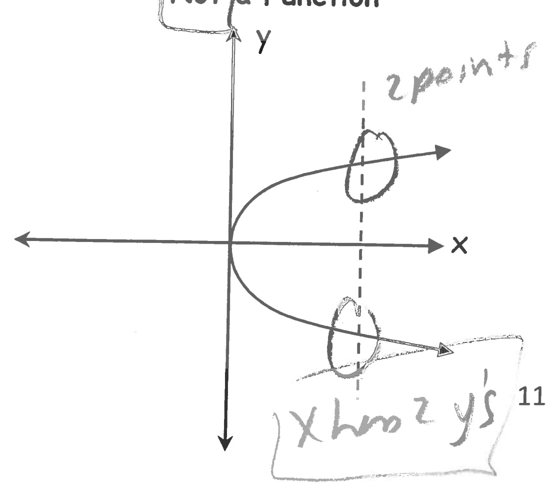
- To determine whether a **graph** is a function or not, we may apply the straight line test. Since a function has only *one y-value for* every *x-value*, a straight line will cross the graph of a function in no more than one point.

- If the *vertical line* crosses the graph of a relation in more *than* one *point*, the relation is NOT a function.

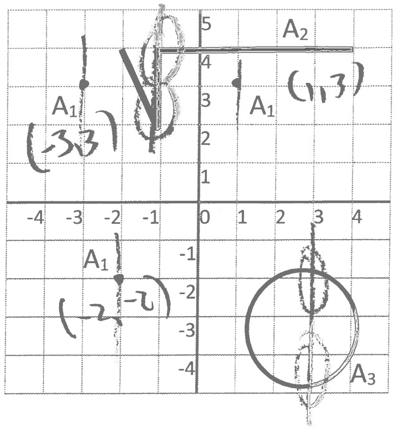
Function

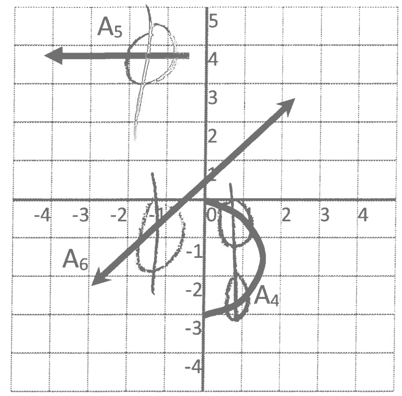


Not a Function



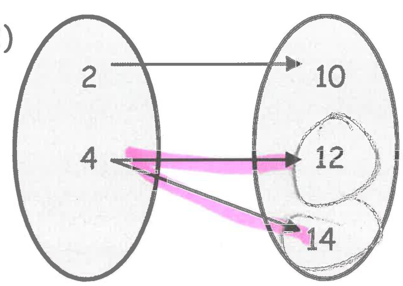
Are these functions?





1) A_1 ? yes A_2 ? NO A_3 ? NO A_4 ? NO A_5 ? yes A_6 ? yes

2)

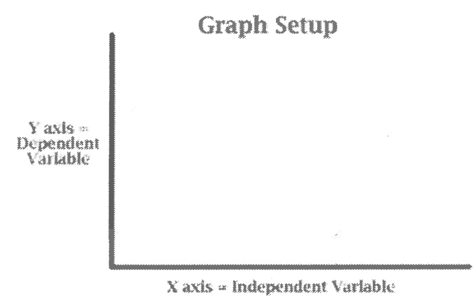


no.

3)

x	0	2	-2	4	-3
y	1	5	-3	9	-5

yes.



Dependent and Independent Variables

In any given model or equation, there are two types of variables:

- Independent variables - The values that can be changed in a given model or equation. They provide the "input" which is modified by the model to change the "output." It is not affected by the other variable. ***It is the "cause"***. It is usually placed on the horizontal, or x-axis.

For example, someone's age might be an **independent variable**. Other factors (such as what they eat, how much they go to school, how much television they watch) aren't going to change a person's age. In fact, when you are looking for some kind of relationship between variables you are trying to see if the independent variable causes some kind of change in the other variables, or dependent variables

- Dependent variables - The values that result from the independent variables. The dependent variable **depends** on the value that is input. It is affected by the change in the independent variable. Its value is determined by the choice of independent variable. ***It is the "effect"***. It is usually placed on the vertical, or y axis. It is the 'out put'!

(see example on next page)

(dependent variable, continued)
For example, a test score could be a **dependent variable** because it could *change depending on several factors such as how much you studied, how much sleep you got the night before you took the test, or even how hungry you were when you took it.* Usually when you are looking for a relationship between two things you are trying to find out what makes the dependent variable change the way it does.

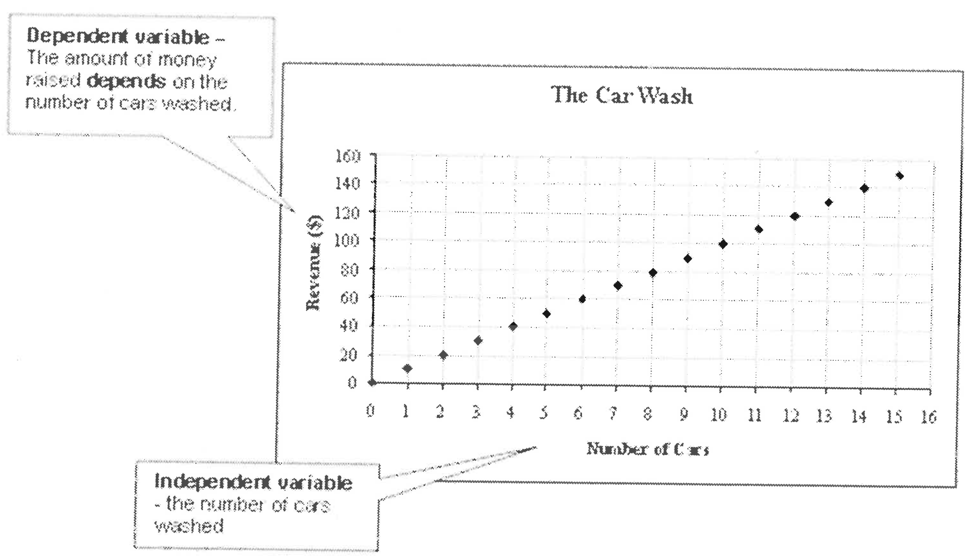
Many people have trouble remembering which is the independent variable and which is the dependent variable. An easy way to remember is to insert the names of the two variables you are using in this sentence in the way that makes the most sense. Then you can figure out which is the independent variable and which is the dependent variable:

(Independent variable) causes a change in (Dependent Variable) and it isn't possible that (Dependent Variable) could cause a change in (Independent Variable).

For example:

(Number of cars washed) causes a change in (revenue $ [*the amount earned*]) and it isn't possible that (revenue) could cause a change in (# of cars washed).

We see that "number of cars washed" must be the independent variable and "revenue" must be the dependent variable because the sentence doesn't make sense the other way around.



As the number of cars goes up, the revenue goes up.

We can think of a function as an input/output machine. **The input can be any number in the domain** (the *horizontal value on the x-axis*) and the **output** (the *vertical value on the y-axis)* **depends on the input.** So the domain is the independent variable and the output is the dependent variable.

In a relation, the input is the **independent variable**. The output is the **dependent variable**, because the value of the output depends on the value of the input.

Relation

$$y = -3x + 5$$

↑ ↑

Dependent Variable, y (Output) — Independent Variable, x (Input)

When a relation is shown as a graph, the independent variable is shown on the horizontal axis and the dependent variable is shown on the vertical axis.

The **domain** of a relation is the set of all the possible values for the independent variable. The **range** of a relation is the set of all the possible values for the dependent variable.

example, if $C = 15 + 2n$, this notation shows that C is dependent variable as it depends on n.

Try these:

1. The table shows masses of different numbers of Canadian quarters.

The mass of quarters, m, depends on the number of quarters, n.

We say that the mass is the dependent variable and the number of quarters is the independent variable.

Domain: {1, 2, 3, 4, 5}

Range: {4.4, 8.8, 13.2, 17.6, 22.0}

Is the relation also a function? Why? Yes / no because every 'number of quarters' has only one 'mass'

ind Number of Quarters, n	dep Mass, m (g)
1	4.4
2	8.8
3	13.2
4	17.6
5	22.0
dom	range

2. This table shows sample costs for a pay-as-you-go cell phone plan.

Number of Minutes, n	Cost, C ($)
10	2
20	4
30	6
40	8
50	10

domain range

a) Is this relation also a function? yes

number of minutes cost

b) Identify the independent and dependent variables.

c) Write the domain and range. {2, 4, 6, 8, 10}

{10, 20, 30, 40, 50}

Function Notation

We can write an equation that represents a function, using FUNCTION NOTATION.
Any function that can be written as an equation in two variables can be written in function notation.

→**Example**:

The revenue is a function of the number of cars washed. The equation **R = 10 n** represents the revenue.

When the input is n, number of cars, the revenue, R, ~~in dollars is R = 10n~~. Since R is a function of n, we can write an equation that represents this function using ***function notation.***

To show R = 10n is a function, we describe or write it in function notation: **R (n) = 10 n**

We say: *R depends on n* *"R of n"*

R of n is equal to 10 times n or 10n

This notation shows that **R** is the **dependent variable** and that **R depends on n.**

R (6) represents the value of the function when n = 6. It represents the revenue when there are 6 cars washed.
R (n) = 10n
R (6) = 10 (6)
R (6) = 60
So the revenue when 6 cars is washed is $60.

Substitute 6 in for n.

a) Determine or find R (5).

$R(n) = 10n$
$R(5) = 10(5)$
$R(5) = 50$

This number represents that when 5 cars are washed, the revenue is $ 50.

b) Determine or find the value of n when R (n) = 80. What does this number represent?

$R(n) = 10n$
$\frac{80}{10} = \frac{10n}{10}$
$8 = n$

When the revenue is $80, 8 cars have been washed.

Try these: Carmen works for a research company in a shopping mall.
The equation $P = 5n + 30$ represents her daily pay, P dollars, when she conducts n surveys.

a) Describe the function. Write the equation using function notation.

The pay Carmen receives is a function of the number of surveys she conducts.

$P(n) = 5n + 30$

b) Find the value of P(8). What does this number represent?

$n = 8$

$P(8) = 5(8) + 30$
$= 40 + 30$
$= 70$

$70 is the pay she receives after conducting 8 surveys

c) Find the value of n when P(n) = 90. What does this number represent?

$P(n) = 5n + 30$
$90 = 5n + 30$
$60 = 5n$
$12 = n$

$90 = 5n + 30$
$-30 \quad -30$
$60 = 5n$

12 is the number of surveys she conducts to earn $90.

Example 2: Write in function notation.

a) $P = 2s + 15$

$P(s) = 2s + 15$

b) $y = -3x + 5$

$y(x) = -3x + 5$

Write as an equation in 2 variables.

a) $d(t) = 4t - 7$

$d = 4t - 7$

b) $g(x) = -2x - 3$

$g = -2x - 3$

Example 3: Given $f(x) = x^2 - 6$

a) Find $f(0) = -6$

$f(0) = (0)^2 - 6$
$f(0) = -6$

b) Find $f(10) = 94$

$f(10) = (10)^2 - 6$
$f(10) = 94$

c) Find x if $\boxed{f(x)} = 30$ $\boxed{x = 6}$

$30 = x^2 - 6$
$36 = x^2$
$6 = x$

Practice 5.2 p. 270-272 #1,3,4,5,9,14,18. 15 (14a+b) and #8 p.244

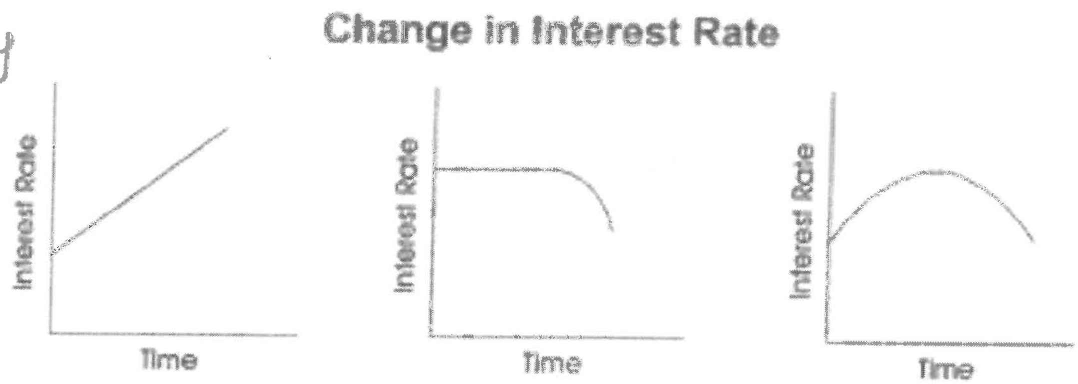
Quiz 5.1 5.2 Tues Ap 3 - checkpoint paper - review 5.1 & 5.2 test corrections catchup ch 3+4

only vertical line (not domain / range) test signed + sealed

5.3 INTERPRETING AND SKETCHING GRAPHS

The shape of a graph can tell us a great deal even if there are no numbers on the axes. For example, the set of graphs below show how interest rates have changed over one year in three different countries.

interpreting graphs from shape



The first graph shows that interest rates in that country rose steadily throughout the year.

stay the same

The second graph shows that interest rates were high at the beginning of the year, remained constant for most of the year, and dropped rapidly for the last part of the year.

The third graph shows that interest rates increased rapidly at first, then more slowly to reach a maximum about halfway through the year, and decreased for the rest of the year.

Hints for interpreting graphs:

- When the value of the independent variable is zero, what is the value of the dependent variable? Does it make sense?
- As the value of the independent variable increases, does the value of the dependent variable increase or decrease? What does this mean for these particular variables? Does it make sense?
- Does the value of the dependent variable change at a steady rate? If not, how does it change? Is the change faster at first and slower later on . . . or is it slow at first and faster later on?

"A picture is worth a thousand words." Graphs are a picture of data. In this section you will investigate how to create and interpret the "story" graphs are telling.

- Speed represents the change of distance over time
- Speed = d/t
- Velocity is speed with a direction
- A negative velocity indicates a movement in the opposite direction. Slope = Speed (m= "rate" of speed)

Walking Slowly (Least steep)

Walking Normally

Walking Quickly (Steepest)

Stops m = 0 horizontal

Positive Slopes have lines that are in an upward direction

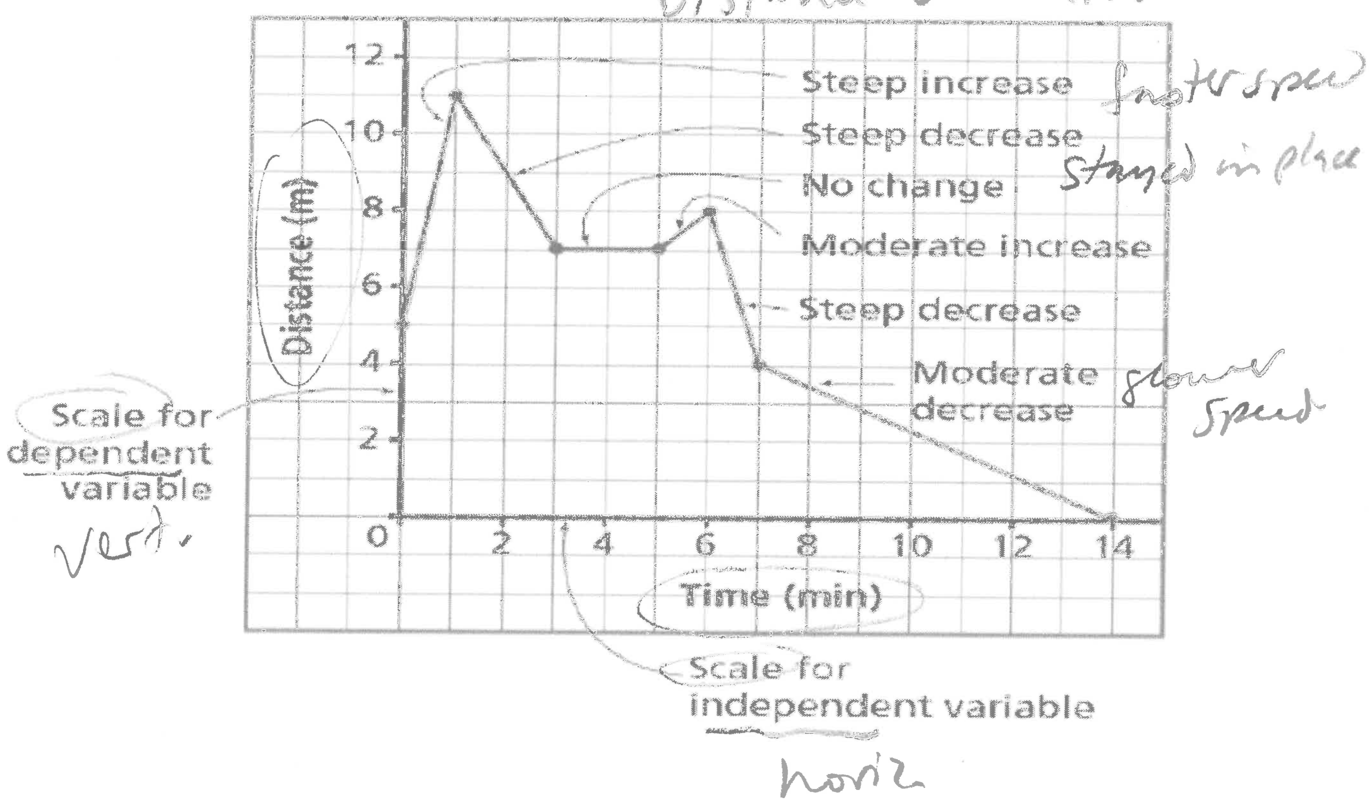
Negative Slopes have lines that are in a downward direction

Graphs can provide much information

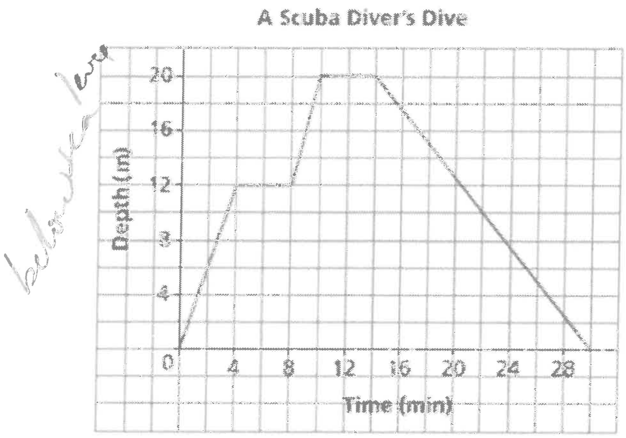
- ☐ Look at title, and axis labels to gather as much info as possible
- ☐ Think of realistic reasons for what is happening in the graph

Properties of a Graph

Distance over Time



Example 1: Consider the following graph:



a) How many minutes did the dive last? 30

b) At what times did the diver stop her descent?
4min, 10min 14-30 ascent
4-8 10-12

c) What was the greatest depth the diver reached? 20m For how many minutes was the diver at that depth?
4min.

Example 2: ←

When interpreting a broken-line graph, keep in mind what information is being displayed on the axes. For example, the graph below shows the journey taken by Little Red Riding Hood on her motor scooter from her home to her grandma's house. More specifically, the graph shows the distance (in kilometres) of Little Red Riding Hood from her home over time (in minutes). It also shows the speed at which she travelled along her route. ←(velocity)

a. Write a brief description of the situation depicted in the graph.
Little Red Riding Hood walks to grandmother's house, stays a while, then comes home. But her speed increases and decreases both ways and sometimes stops (picking flowers? talking to wolf?)

b. How long did her journey take in total? ⇨ 110 m

c. Where did she end up at the end of her travels? ⇨ home

d. At what time did she reach her grandma's house? ⇨ 50 mins

e. How long did she stay at her grandma's house? ⇨ 20min

f. How many times did she stop on her journey? ⇨ 5 (incl. at grandma's)

$V = \frac{D}{T}$ 18-8=10 20

g. What was Riding Hood's speed between 30 and 50 min? velocity ⇨ 0.5 km/1min

$V = \frac{10}{20} = 0.5$ km/h.

h. When was she travelling at the fastest speed? ⇨ 95-100 min

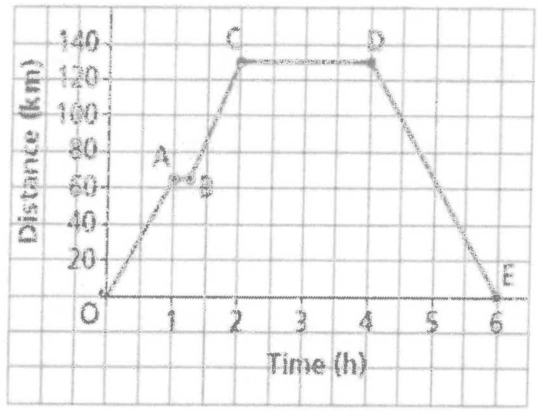
i. During the first half of her journey, the slopes are mostly positive (ie. go up), whereas the slopes for the second half are negative mostly negative (ie go down). What does this mean? distance from her house is decreasing

positive $\frac{rise}{run} = \frac{1}{1} = 1$ time ↑ distance from home on way there ↑

time ↑ distance from home on way home ↓ $\frac{rise}{run} = \frac{-1}{1} = -1$

Example 3: This graph represents a day trip from Winnipeg to Winkler. The distance is approximately 130 km.

Day Trip from Winnipeg to Winkler, Manitoba

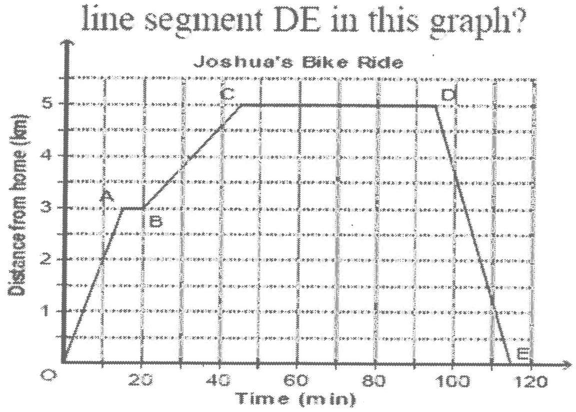


The distance between Winnipeg and Winkler is 130 km.

a) Describe the journey for each segment of the graph.

segment	graph	journey
$\overline{OA}$	*The graph goes up to the right. As time increases, the distance from Winnipeg increases.*	*Travelled 65 km in first hour of trip.*
$\overline{AB}$	The graph is horizontal.	stopped 15 mins. (break?)
$\overline{BC}$	Graph goes up to right. As time increases, distance from Winnipeg increases	14.-65 back on road ~~65~~ km. 45 min
$\overline{CD}$	The graph is horizontal.	stopped 2 hrs. (visit)
$\overline{DE}$	The graph goes down to right. As time increases, distance from Winnipeg decreases.	returned home 2h no stops. 130 km 65 km/h.

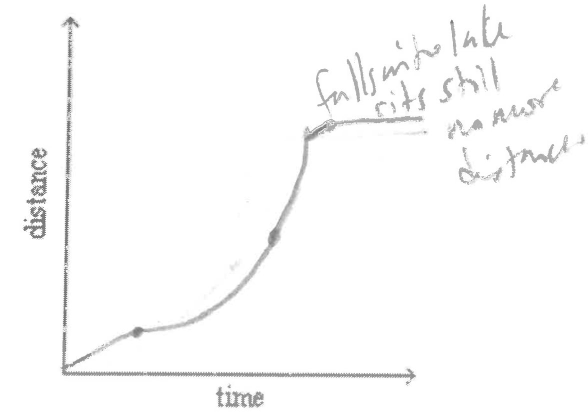
Example 4: Joshua went on a bike ride. Which statement best describes what is happening for line segment DE in this graph?



a) Joshua spends time at the park. b) Joshua leaves home. c) Joshua cycles to the park. d) Joshua returns home.

5. Draw the graph that represents the following:

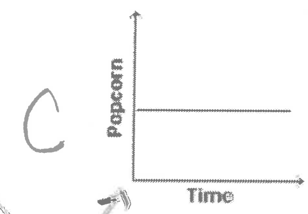
- A log floats in a slow, steadily moving stream.
- It goes through 2 sets of rapids; the second is faster than the first.
- It then goes over a waterfall into a lake.



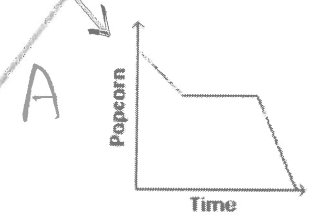
6. Match each situation to one of the graphs.
Use a ruler to draw a straight line from the situation to the correct graph.

(Or write letter of situation beside graph.)

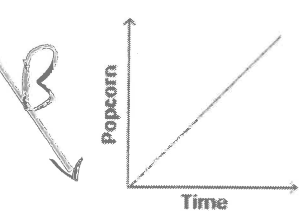
(a) Melissa eats her popcorn for a short time.
She stops eating to speak to her friend Sam.
Then she finishes eating her popcorn very quickly.



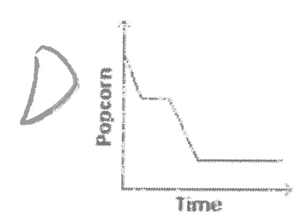
(b) Melissa makes popcorn at a constant rate before the movie



(c) Melissa does not eat any of the popcorn she has made.

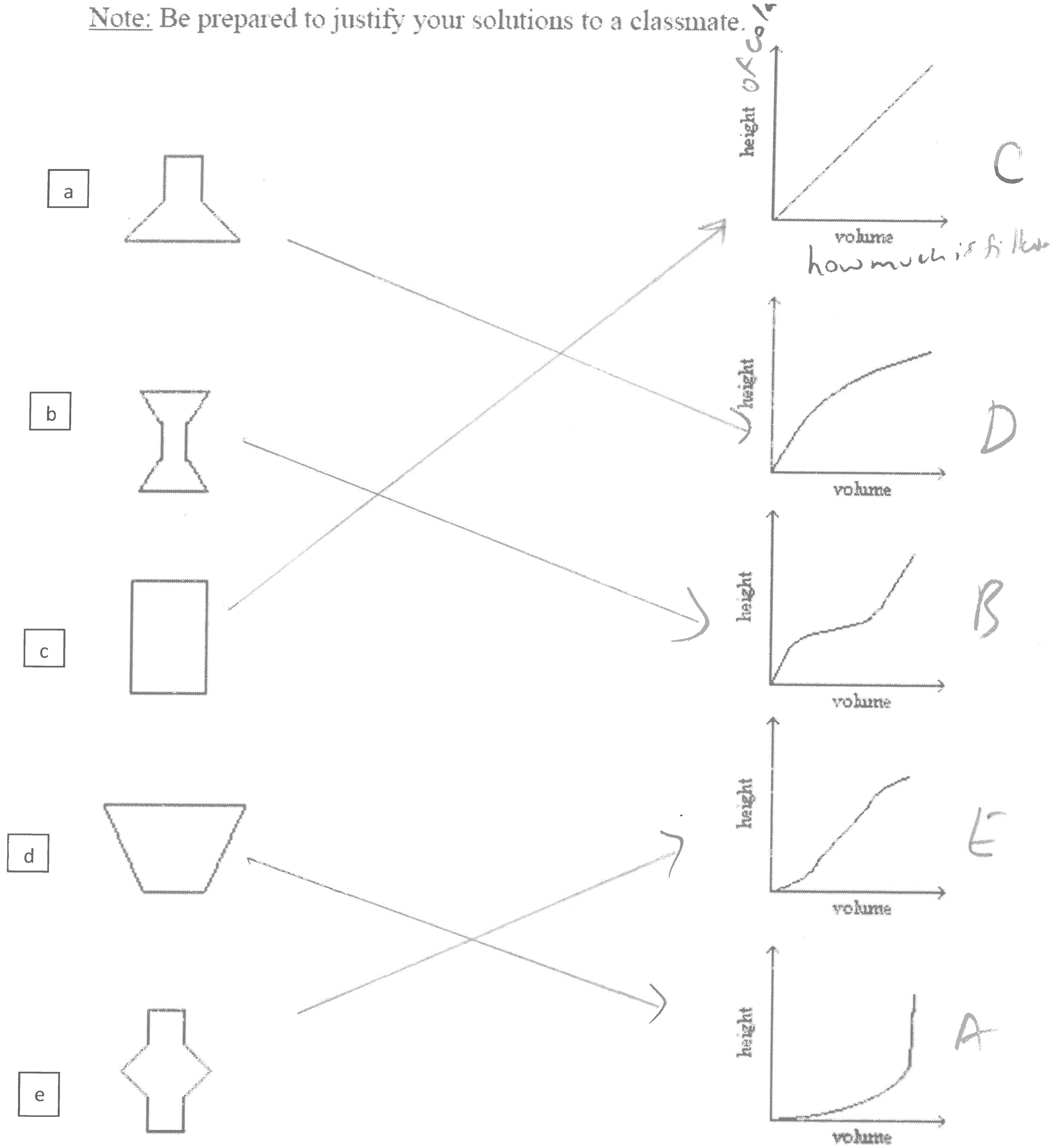


(d) Melissa eats her popcorn quickly for a short time.
She stops eating during the intermission of the movie.
She begins eating again when the movie starts, but she does not finish the popcorn because she is feeling full.



7. Imagine that you are pouring cola into each of these glasses at a constant rate. Match eacl glass to the graph that best represents the filling process. *Use a ruler to draw a straight line, the situation to the correct graph.* (Or write letter of situation beside graph.)

Note: Be prepared to justify your solutions to a classmate.

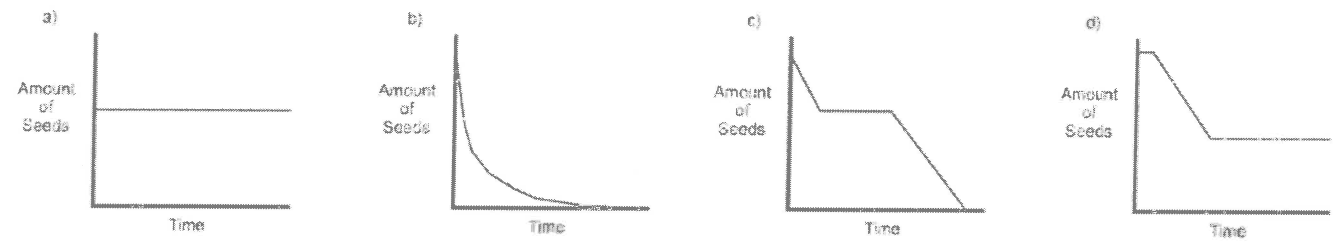


Try these

1. Sunflower Seed Graphs

Ian and his friends were sitting on a deck and eating sunflower seeds. Each person had a bowl with the sa amount of seeds. The graphs below all show the amount of sunflower seeds remaining in the person's bov over a period of time.

Write sentences that describe what may have happened for each person.



a) didn't eat them

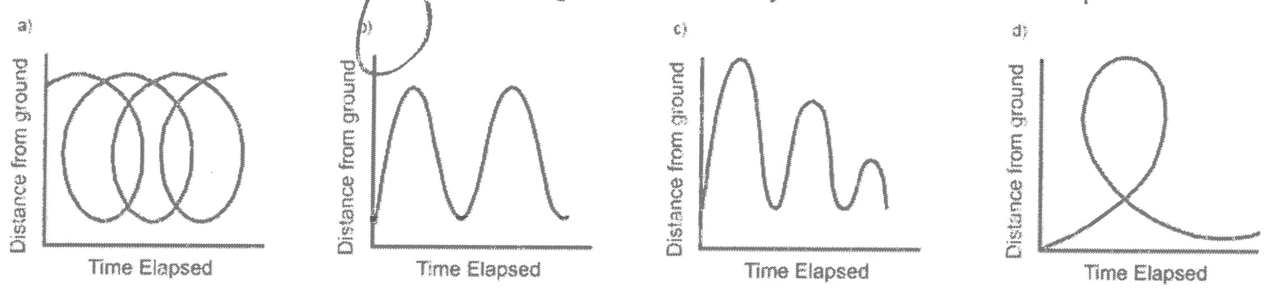
b) ate a lot quickly then more slowly until gone

c) ate. stopped. ate again until gone.

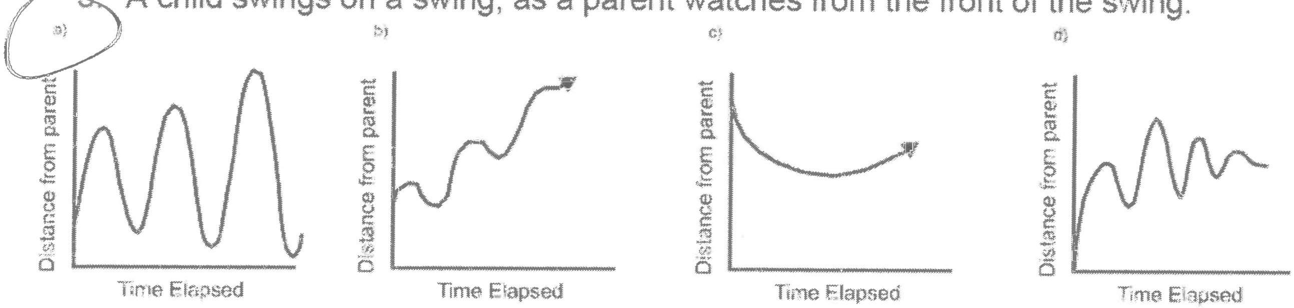
d) waited. ate. stopped. (not all gone.)

Indicate which graph matches the statement. Give reasons for your answer.

2. A bicycle valve's distance from the ground as a boy rides at a constant speed.



3. A child swings on a swing, as a parent watches from the front of the swing.



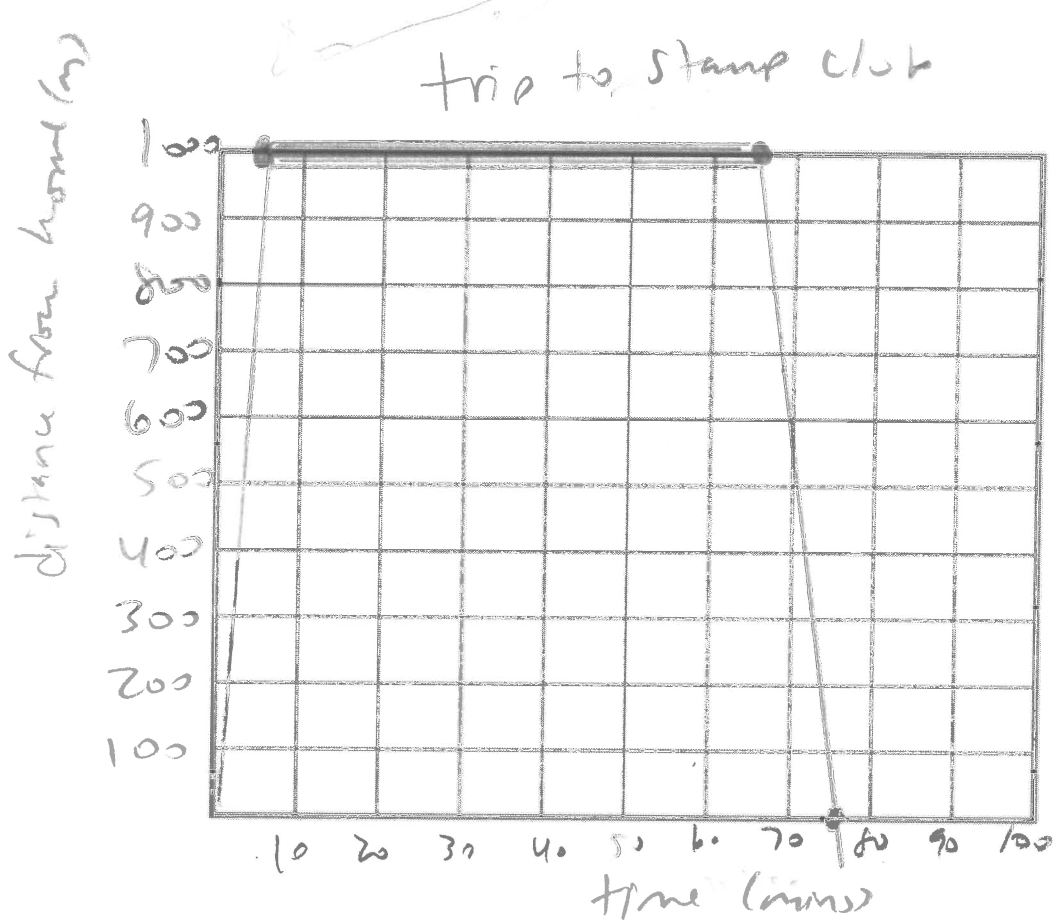
3. You are at home getting ready to go out to your stamp collecting club.

You leave your house and jog the 1000m to the club. You arrive 5 minutes later.

You exchange stamps and chat for 1 hour, then leave for home. It takes you 10 minutes.

Plot a distance time-graph to represent your journey to and from the club.. a graph of speed as a function of time. Label each section of your graph, and explain what it represents.

(Don't forget: Choose a suitable scale for each axis. Decide how many points to plot. Draw the graph with suitable accuracy. Provide a title and label each axis. Give your graph a title, label each axis, and choose an appropriate scale for each variable.)



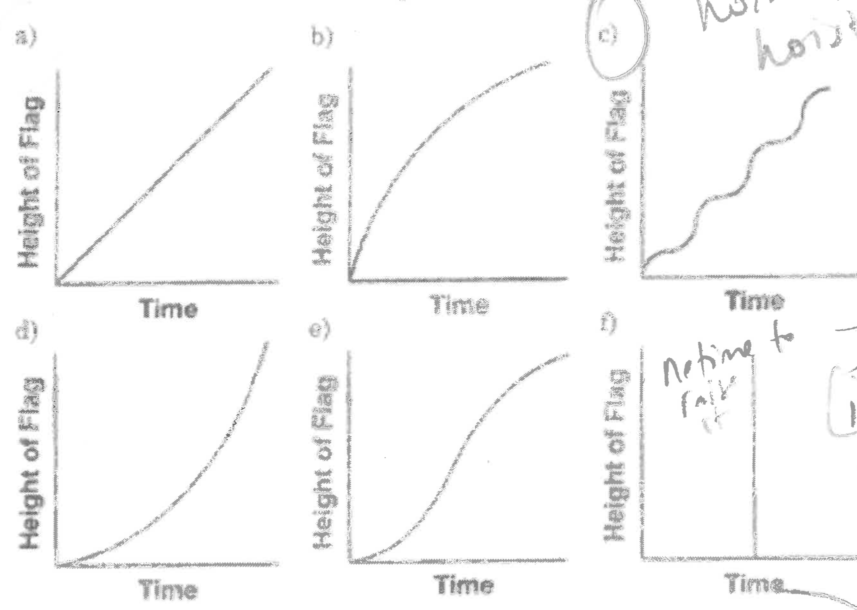
5.3 practice p. 281 – 283 (~~Some questions can be answered on reverse~~. You will need graph paper for one of the questions.) #1,4,5,8, 10, 13, 15

Extra practice 5.3

1: Every morning at camp, one of the scouts hoists a flag to the top of a flagpole. The graphs below show the height of the flag as a function of time. Which do you think models the situation most realistically? If you think none of the graphs is realistic, draw your own version and explain it. Do any of the graphs represent an impossible situation?

hoist regrip
hoist regrip

Flag Hoisting



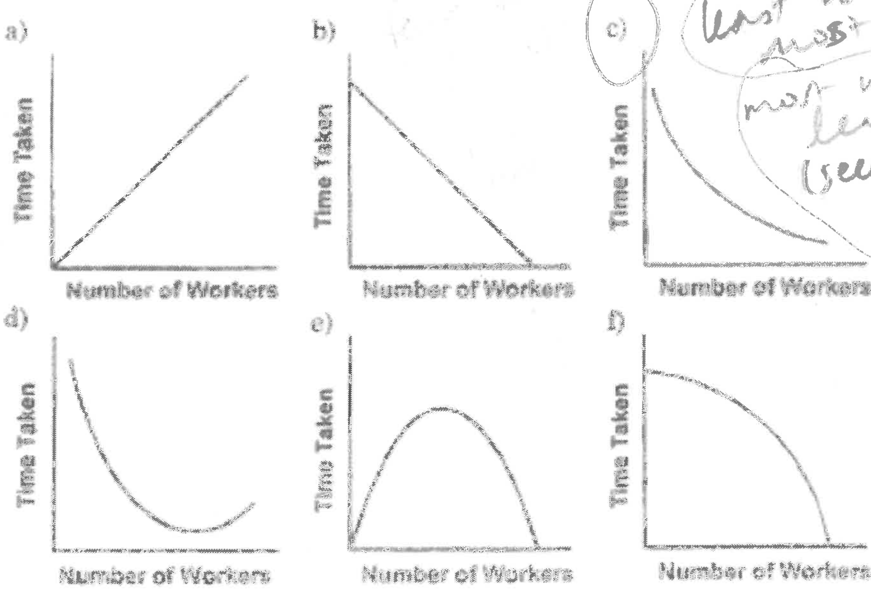
impossible ex time is 3 seconds and somehow flag jumps to top and it didn't take 1 sec to see etc.

must start with 0 height.

2: A group of city workers have to plant a large number of seedlings, Which of the following graphs models most realistically the relationship between the number of workers involved and the time it takes to complete the job? Explain your answer.

p.54

Planting Seeds



least workers most time

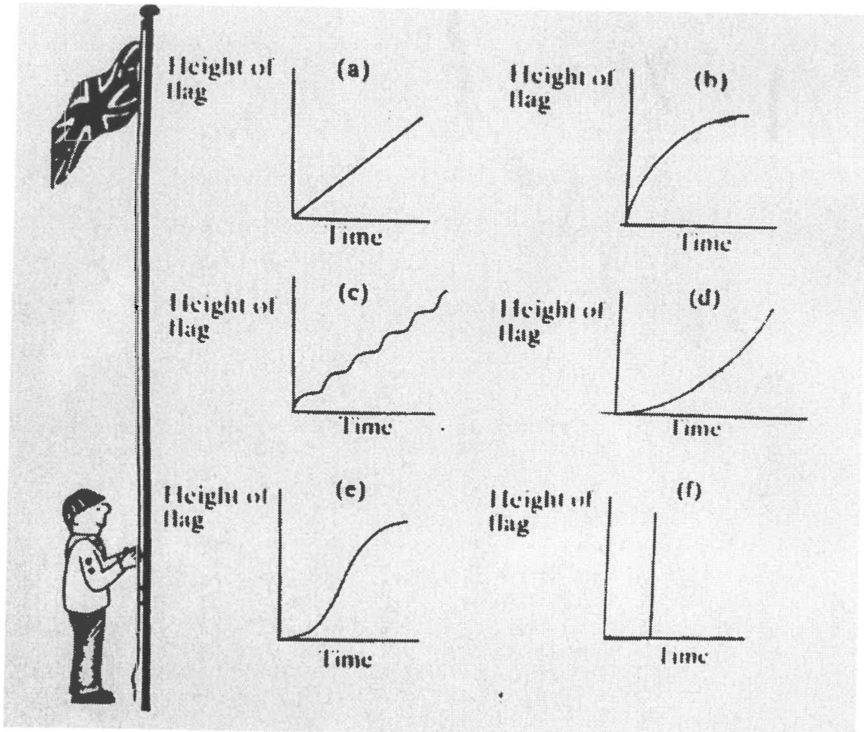
most workers least time. (seedlings not planted by 0 workers)

double time 1 people to plant 3 people

graph cannot meet either axis

Can never be so many people that it takes 0 minute.

extra practice 5.3 #1



Picture

Hoisting the Flag (Blog Post)

Every morning, during summer camp, the youngest boy scout has to hoist a flag to the top of a flag pole.

Explain in words what each of the graphs above would mean.

Graph (a) indicates that the height of the flag would be affected because it would be hoisted at a 'constant rate'.

Graph (b) indicates that the flag would be hoisted at a quicker rate but gradually become slower.

Graph (c) indicates that the flag would be hoisted at a distinct yet reoccurring rate of height/time, because upon hoisting the flag, it would take a moment to get another grip around the rope to pull it up again.

Graph (d), which is the opposite of Graph (b), indicates that the flag would be hoisted at a slower rate but gradually become faster.

Graph (e), is the combination of Graphs (b) & (d), which indicates that the flag would be hoisted at a quicker rate then gradually become slower and then become hoisted at a slower rate and gradually become faster.

Graph (f) indicates that the flag would require no time to raise it to the specific height which is impossible.

Which graph shows this situation most realistically? Explain.

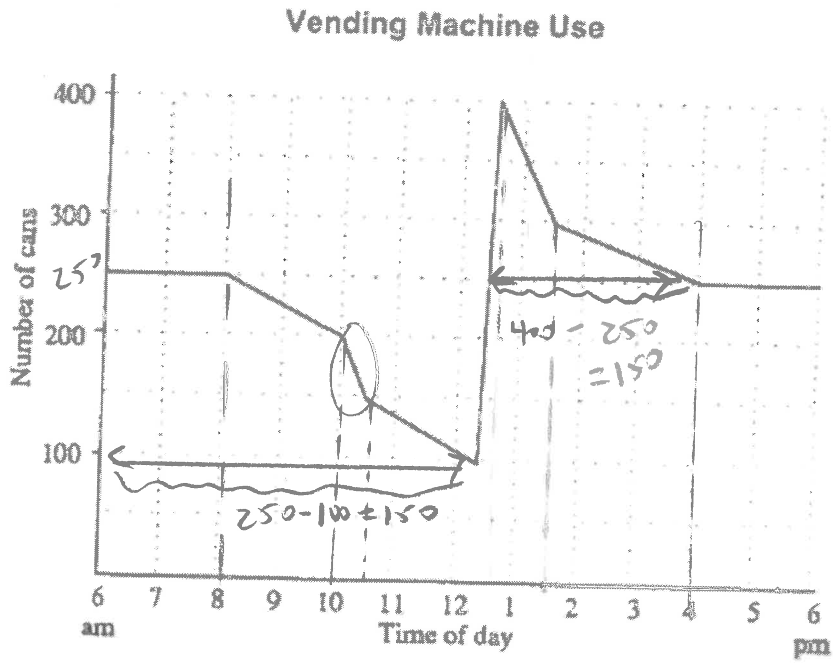
Personally, I believe that Graph (c) would be the most realistic because of the given definition; the flag would be hoisted at a distinct yet reoccurring rate of height/time, because upon hoisting the flag, it would take a moment to get another grip around the rope to pull it up again.

Which graph is the least realistic? Explain.

Graph (f) because it indicates that the flag would require no time to raise it to the specific height which is impossible. (That I know of.)

(25)

3: A company cafeteria has a large vending machine that sells cans of soft drinks. The graph below shows the number of cans in the machine on a typical day.



a) Describe how the number of cans in the machine varies during the day.

Cans decrease gradually all day but decrease more rapidly at break & lunch

b) When are morning coffee breaks and lunch-times?

10-1030 sharp decrease in cans in machine (more people buying)

1230-130 lunch

c) What happens just before lunch-time?

machine is refilled

d) Can employees use the drink machine during working hours?

Yes because cans gradually decrease all day.

e) How many cans of drinks were sold during this day?

150 + 150 = 300

f) What appears to be the working hours for this company? How do you know?

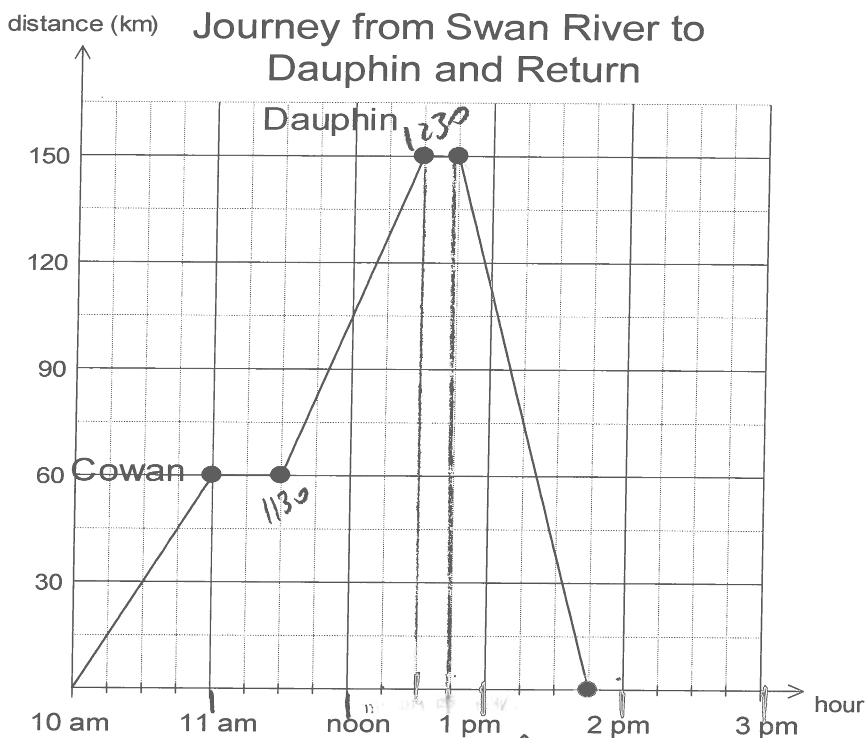
8 - 4

none sold 6 - 8

none sold 4 - 6

4

- The graph shows a journey by car from Swan River to Dauphin and return.



60 ÷ 5 = 12

a) How far is it from Swan River to Cowan?

60 km

b) How far is it from Cowan to Dauphin? 150 - 60 = 90 km

c) At which two places does the car stop? Cowan + Dauphin

d) How long does the car stop at Dauphin? 1230 - 1245 15 min.

e) At what time does the car

i) arrive in Cowan? 11 AM

ii) arrive back in Swan River? 1:45

f) At what speed is the car travelling

i) from Swan River to Cowan? 60 km in 1 hr. 60 km/h.

ii) from Cowan to Dauphin? 150 - 60 = 90 km in 1 hr. 90 km/hr.

1130 - 1230

iii) from Dauphin to Swan River?

1245 - 145 1 hr.

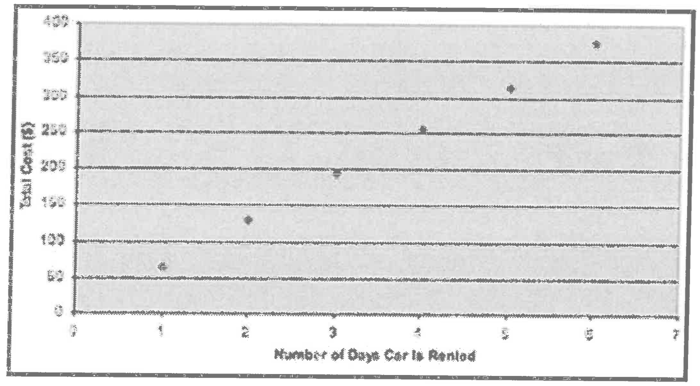
150 km 1 hr.

27

5.4 p. 284 Graphing Data

To rent a car for less than one week from Ace Car Rentals, the cost is $65 per day for the first three days, then $60 a day for each additional day.

Number of Days Car Is Rented	Total Cost ($)
1	65
2	130
3	195
4	255
5	315
6	375



Why are the points on the graph not joined? each day has a cost (no partial costs for partial days)

Is this relation a function? How can you tell?
Yes. For every Day there is only one cost.

What is the domain? What is the range?

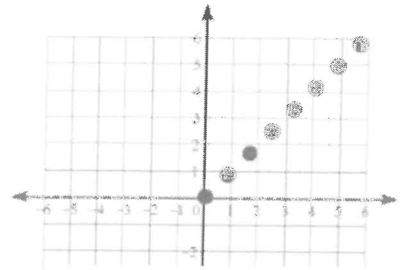
domain {1, 2, 3, 4, 5, 6}

no units range {65, 130, 195, 255, 315, 375}

The graph of a relation can be either ***discrete** or **continuous.*** Where the values between graphed points have meaning in the context of the question, the data is **continuous**. In this case, **it would be appropriate to join the points with a line or a smooth curve.** For example, if you constructed a graph relating the height of a growing plant over time, you would get a continuous graph. *This is because all values of both height and time are meaningful. It is possible to have a height of 10 cm, 11 cm, and all values between 10 cm and 11 cm.*

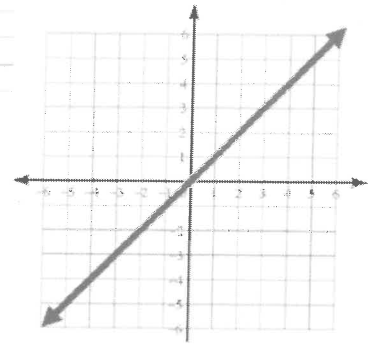
On the other hand, where the values between points have no meaning in the question's context, the data is **discrete**. **It is not appropriate to join the points on a discrete graph.** Discrete data include those **data you can count**. For example, if you graphed the number of people in a store over a period of a business day, you would get a discrete graph. *This is because not all real values are meaningful in this context. While it is possible to have 10, 11, or 12 people in the store, you cannot have 10.5 or 11.777 people in the store!*

Discrete Data



- Data values on a graph that are not connected
- Discrete graph can only show points, because the values in between them have no meaning
- (example shows only 8 ordered pairs of data points →)

Continuous Data



- Data values on a graph that are connected
- Continuous graphs are a solid line or curve
- (example shows Infinite number of possible ordered pairs →)

Example : A popular event at Les Folies Grenouilles is the fireworks display. Assume that the event organizers send off 20 fireworks shells each minute.

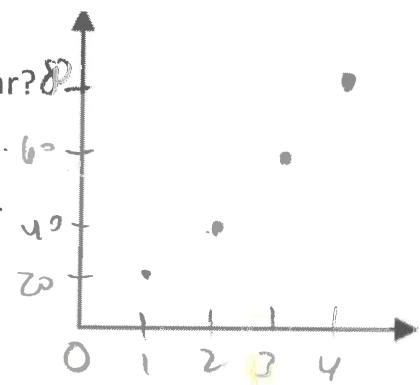
a) Assign a variable to represent each quantity in the relation. Which variable is the dependent variable? Which is the independent variable?

b) Create a table of values for this relation. What are appropriate values for the independent variable? (The independent variable goes in the first column of the table.)

# of minutes, m	# of fireworks, f
1	20
2	40
3	60
4	80

c) Is the relationship between the total number of fireworks and the duration of the event linear or non-linear? Explain how you know. Difference constant for both variables.

d) Create a graph for the relation. Are the data discrete or continuous? Why?



discrete-
no fireworks in between minutes.

Try this

1. a) This table shows the refund, r dollars, for different numbers of juice tetra paks, n. Is this relation a function? Explain. Yes. Each number of tetra paks has only one refund.

   b) Suppose you were to graph the data in this table of values. Would you join the points? Justify your answer.

Number of Juice Tetra Paks, n	Refund, r (\$)
5	0.25
12	0.60
17	0.85
24	1.20
30	1.50

Yes. There can also be 1, 2, 3, 4, (etc.) tetra paks which would receive a refund. Although you can't receive a refund on half a tetra pak.

2. a) Suppose you were to graph the data in this table of values. Would you join the points? Justify your answer. Yes. Times + temperatures exist between those values.

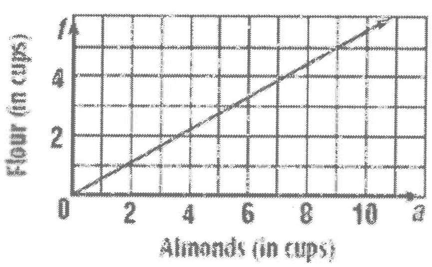
   b) Is this relation a function? Explain.

Time of day (24 hr clock)	Temperature (°C)
08:00	5
11:00	7
14:00	10
17:00	11
20:00	7
23:00	6

Yes. Each time of day has only one temperature.

3.
The relationship between the amount of flour and the amount of almonds required in an almond cake recipe can be represented by the function $\boxed{f = \frac{5}{9}a}$ where a is cups of almonds and f is cups of flour.

ALMOND CAKE RECIPE



Which is the **best** description of the function?

F. a continuous function with a domain of non-negative real numbers

G. a continuous function with a domain of real numbers between 0 and 10

H. a discrete function with a domain of non-negative integers

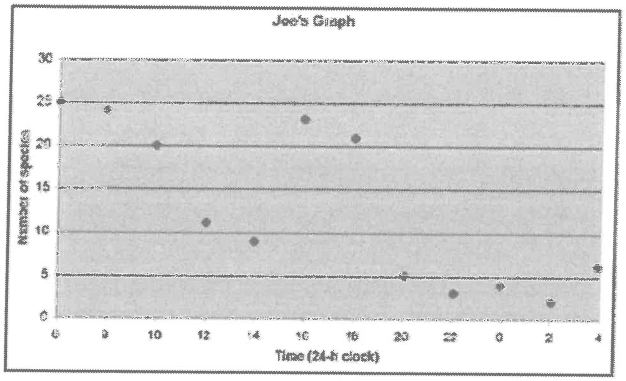
I. a discrete function with a domain of integers between 0 and 10

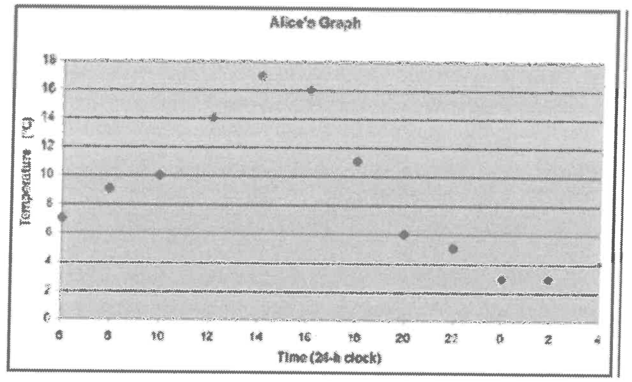
Do question p. 286 #1 *(quick graph sketches – don't need to be perfect)*

5.5 Graphs of functions and Relations p. 287

Here is a good example of discrete and continuous data:

In an environmental study in Northern Alberta, Joe collected data on the numbers of different species of birds he heard or saw in a 2-h period, for 24 h. Alice collected data on the temperature in the area at the end of each 2-h period. They plotted their data:





Every element in domain (F) has only 1 element in range (F).

Does each graph represent a relation? A function? How can you tell?

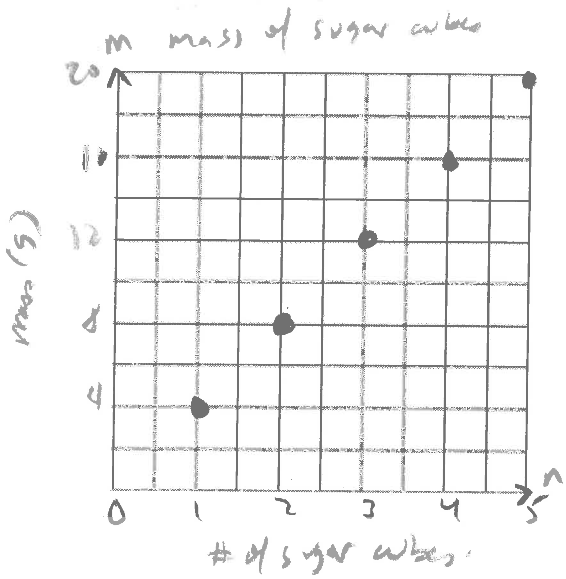
Each is a function because no ~~two~~ points have the same domain. For every point on horizontal axis, there is only one related point on vertical axis.

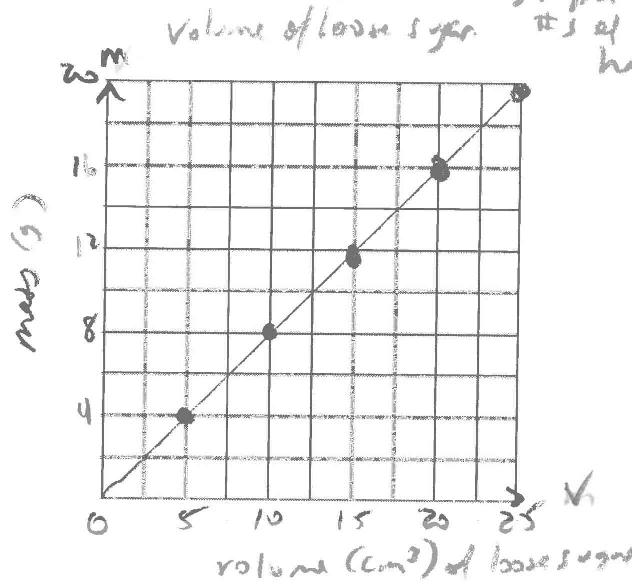
Which of these graphs should have the data points connected? Explain.

Temperature against time – both time and ~~temperature~~ are continuous. Values between data points have meaning.

↓Graph the following (remember to title the axes and the graph itself).↓

Species against time graph – only whole #s of birds. No half birds.





A sugar cube has a volume of 5 cm³ and a mass of 4g. Graph the mass of sugar as a function of the number of sugar cubs from 0 to 5 sugar cubes.

number cubes mass

Five cubic centimetres of loose sugar also has a mass of 4g. Graph the mass of sugar as a function of the volume of sugar from 0 to 25 cm³ of loose sugar.

volume cubes mass

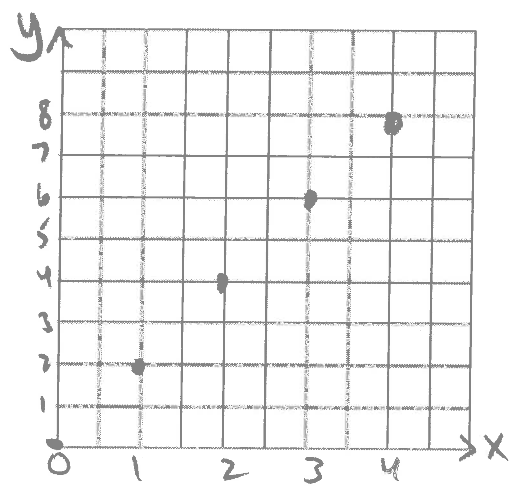
Write the definitions for domain and range (p. 289)

- The domain is defined as the set of independent variables (usually represented by the letter x) or the first component of the ordered pair.
- The range is defined as the set of dependent variables (usually represented by the letter y) or the second component of the ordered pair.

↓Represent the function that associates **every whole number** with its **double** as a table then in a graph.↓

Whole number, x	Double the number, y
0	0
1	2
2	4
3	6
4	8

(the table continues for all whole numbers)



What is the equation of this relation? $y = 2x$

Is the relation $y = 2x$ a function? How do you know? (*Write equation on line.*)

In the table of values, there is only 1 y for every x. In graph, no 2 points lie on same vertical line. Each value of x associates with exactly one value of y. Each ordered pair has a different first element.

Write the domain: Set of all whole numbers and the range: Set of all even whole numbers

How can you tell the domain and range from the graph? Domain is 1st value of ordered pair. Range is 2nd value of ordered pair.

Do we connect the points? Why or why not?

No. The domain and range are discrete values. No values exist between the whole numbers.

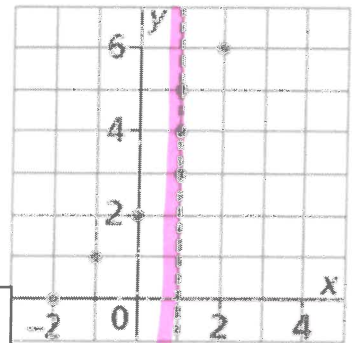
What would happen if this were a different situation, and the domain and range were both all real numbers, how would the graph change? We would draw a line to connect the points. (real numbers include fractions between whole numbers)

Recall:

- A function (only one output value) is always a relation.
- A relation is not always a function.

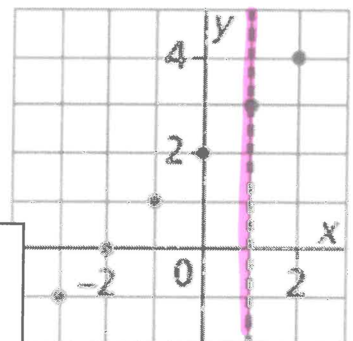
Vertical Line Test for a Function

A relation that is not a function has two or more ordered pairs with the same first coordinate (x value). So, when the ordered pairs of the relation are plotted on a grid, a vertical line can be drawn to pass through more than one point.



A graph represents a function when NO TWO POINTS on the graph LIE ON THE SAME VERTICAL LINE.

A function has ordered pairs with different first coordinates (x value). So, when the ordered pairs of the function are plotted on a grid, any vertical line drawn will always pass through no more than one point.



Place a ruler on a graph then slide the ruler across the graph. If one edge of the ruler always intersects the graph **at no more than one point**, then the graph represents a **function**.

Examples: Look at example 1 p. 290 and then try the following.

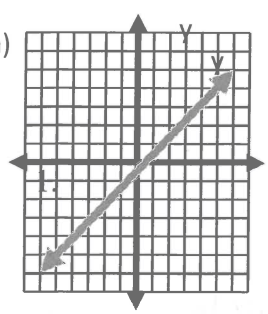
1. Do the ordered pairs represent functions? Why?

a) {(3, 4), (7, 2), (0, -1), (-2, 2), (-5, 0), (3, 3)} No. Ordered pairs have same 1st element. (x)

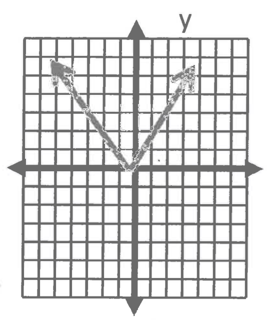
b) {(4, 1), (5, 2), (8, 2), (9, 8)} Yes. Every ordered pair has different 1st element.

2. Do the graphs represent functions? **If not**, show how you know using **vertical line**. Give the **domain** and **range**.

a)

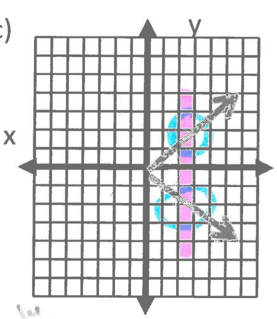


b)



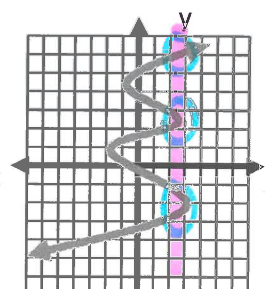
x

c)



x

d)

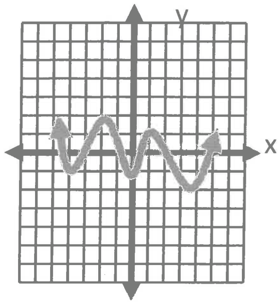


x

x

Function? Y	Function? Y	Function? N	Function? N
D = all real numbers	D = ℝ	D = x ≥ 0	D = x ∈ ℝ
R = y ∈ ℝ "y is element of Real Numbers"	R = y ≥ 0	R = y ∈ ℝ	R = y ∈ ℝ

e)

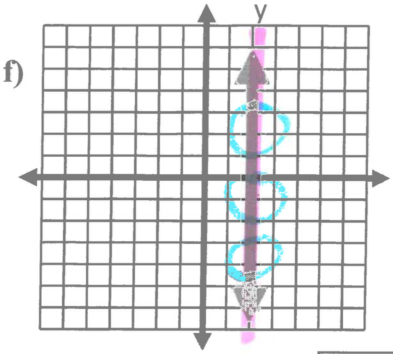


Function? y

D = $x \in \mathbb{R}$

R = $y \in \mathbb{R}$

f)



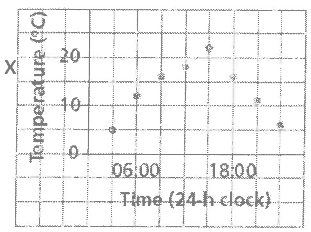
Function? N

D = $X = 2$

R = $y \in \mathbb{R}$

g

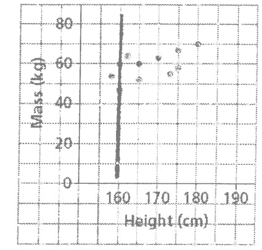
Outside Temperature over a 24-h Period



Function? y

h

Masses of Students against Height

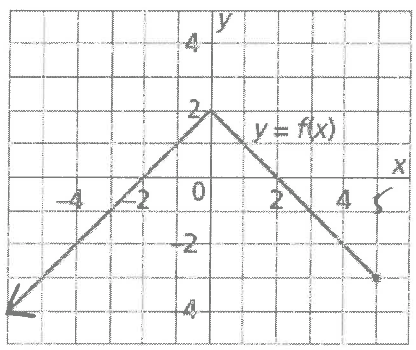


Function? N

Determine the domain and range of the graph of each function.

Look at example 2 p. 291 then try 2a and b.

a)

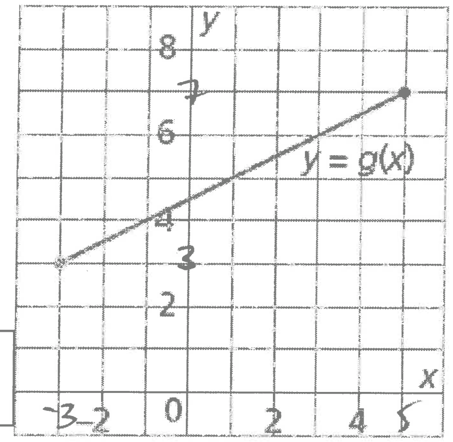


2a)
D = $X \leq 5$

R = $y \leq 2$

Look at example 3 p. 292 then try 3a,b,c ↓

2b



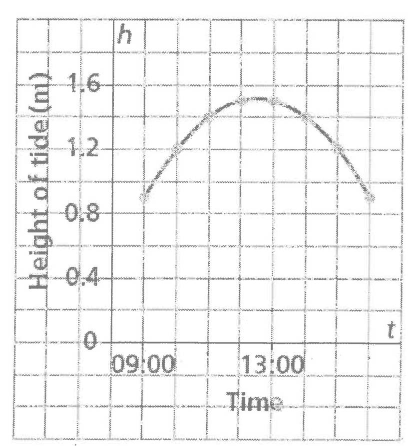
2b)

D = $-3 \leq x \leq 5$

R = $3 \leq y \leq 7$

3. This graph shows the approximate height of the tide, h metres, as a function of time, t, at Port Clements, Haida Gwaii on June 17, 2009.

Height of Tide at Port Clements, June 17, 2009



height

a) Identify the dependent variable and the independent variable. Justify your choices.

time

height depends on time.

b) Why are the points on the graph connected? Explain.

Heights + time between values

c) Determine the domain and range of the graph.

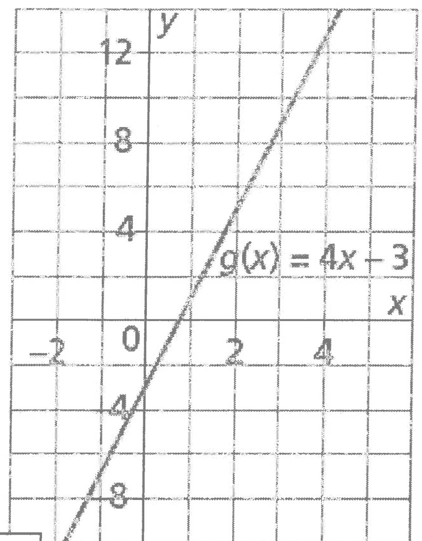
$9{:}00 \leq t \leq 16{:}00$

$0.9 \leq h \leq 1.5$

34

Look at example 4 p. 293 then try 4a,b; 5; 6 ↓

4. Here is a graph of the function
$g(x) = 4x - 3$.



a) Determine the range value when the domain value is 3.

b) Determine the domain value when the range value is −7.

$g(3) = 4(3) - 3$
$= 12 - 3$
$g(3) = 9$

$-7 = 4x - 3$
$+3 \qquad +3$
$-4 = 4x$
$\frac{-4}{4} = \frac{4x}{4}$
$-1 = x$

5 Sandy conducted an experiment with sound waves in dry air at 20°C. She observed that a linear relationship exists between the time and distance that sound travels under these conditions. She recorded her findings in the table below.

DISTANCE SOUND TRAVELS IN DRY AIR AT 20°C

Time (in seconds)	Distance (in kilometers)
4.0	1.372
5.0	1.715
6.0	2.058
7.0	2.401
8.0	2.744

Based on the information in the table, which of the following is a valid statement about Sandy's recorded findings?

A. Sandy's data is discrete with a range of $4 \leq x \leq 8$.

B. Sandy's data is continuous with a range of $4 \leq x \leq 8$.

C. Sandy's data is discrete with a range of $1.372 \leq y \leq 2.744$.

2nd elements

D. Sandy's data is continuous with a range of $1.372 \leq y \leq 2.744$.

↑ numbers between given values

6 Data in a science experiment is related by the function $3x + y = 8$. Carol collected the following x-values in the experiment.

x	-4	0	2
y	20	8	2

What is the range of the function represented by the data?

a. -4, 0, 2
b. -4, 8, 14
c. 20, 8, 2
d. 4, 8, 10

$y + 3(0) = 8$
$y = 8$

$y + 3(2) = 8$
$y + 6 = 8$
$-6 \quad -6$
$y = 2$

$y + 3x = 8$
$y + 3(-4) = 8$
$y - 12 = 8$
$+12 \quad +12$
$y = 20$

DOMAIN AND RANGE OF RELATIONS - notation

[ ] closed dots - includes number $\leq, \geq$

( ) open dots - doesn't include number $<, >$

- There are many different ways of indicating domain and range:

"such that" "is an element of" all Real numbers

Set Notation	**Interval Notation**
a) $\{x \mid x \in \mathbb{R}\}$ (x is part of the set of real numbers)	$(-\infty, \infty)$
b) ______ (x is greater than or equal to 6)	endpoint includes 6 → $[6, \infty)$

$\{x|x \geq 6\}$

$\{x|x \in R, x \geq 6\}$

$x \geq 6$

6 to infinity

can't use parenthesis because we can't equal infinity

c) $\{x \mid -2 < x < 1\}$ (greater than -2 or less than 1) (between -2 and 1) — $(-2, 1)$ endpoint does not include

-2 1

Put in order left to right as appears on number line.

Rules for what brackets to use:

- When simply listing numbers, use <u>squiggly</u> brackets.

Ex: State the domain and range for the following relation: {(1,2), (-2, 3), (3, 4)}

D= $\{1, -2, 3\}$ R= $\{2, 3, 4\}$

- <u>square</u> brackets includes the number. [ ] (Closed dots ●)

"include 3" Ex: $[3, 5]$ $x \geq 3, x \leq 5$ $3 \leq x \leq 5$

3 5

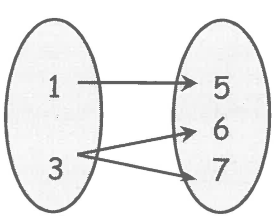
- <u>round</u> brackets does **not** include the number. (open dots ○)

"not 3" Ex: $(3, 5)$ $x > 3, x < 5$ $3 < x < 5$

3 5

Examples: State the *domain* and *range* of the following relation. Also, list the ordered pairs.

1)



D = $\{1, 3\}$ R = $\{5, 6, 7\}$

Ordered pairs:

$\{(1,5), (3,6), (3,7)\}$

2) State the domain (D) and range (R) of the following *ordered pairs*:

a) $A_1 = \{(1,4),(2,5),(3,6)\}$ D = $\{1, 2, 3\}$ R = $\{4, 5, 6\}$

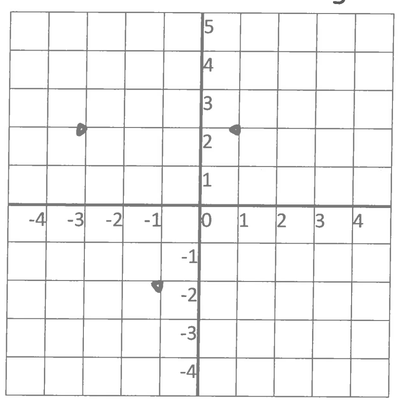
b) $A_2 = \{(0,-1),(0,0),(0,1),(0,2),(0,3)\}$ D = $\{0\}$

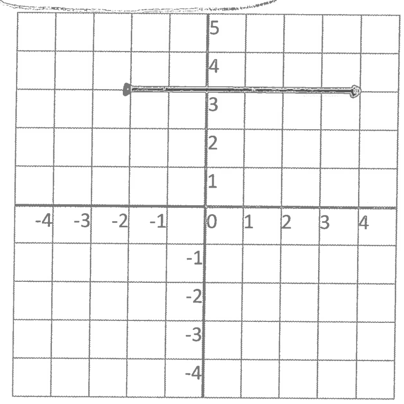
R = $\{-1, 0, 1, 2, 3\}$

c) $A_3 = \{\ldots,(-2,0),(-1,1),(0,2),(1,3),(2,4),\ldots\}$ D = $\{\ldots -2, -1, 0, 1, 2 \ldots\}$

R = $\{\ldots 0, 1, 2, 3, 4 \ldots\}$

3) State the domain and range of the following relations in **interval notation**:



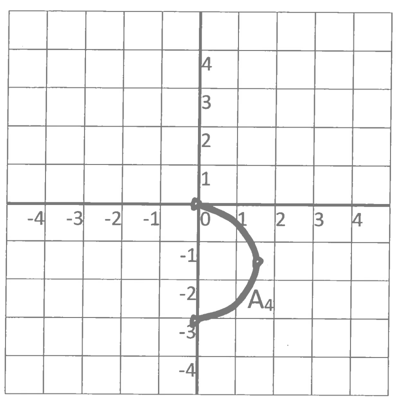


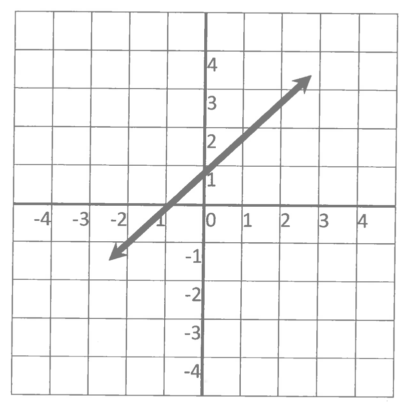
D_1 = $\{-3, -1, 1\}$

R_1 = $\{-2, 2\}$

D_2 = $[-3, 4]$

R_2 = $[3]$



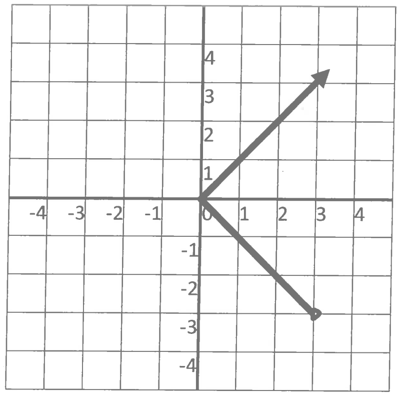


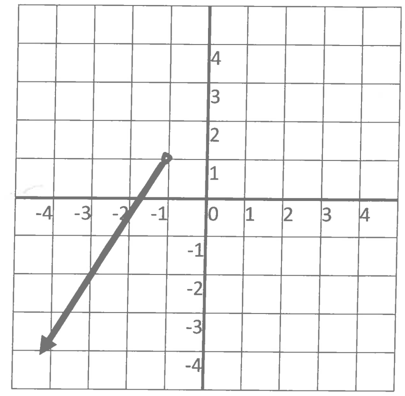
D_3 = $[0, 1.5]$

R_3 = $[-3, 0]$

D_4 = $(-\infty, \infty)$

R_4 = $(-\infty, \infty)$





$D_5 =$ $[0, \infty)$

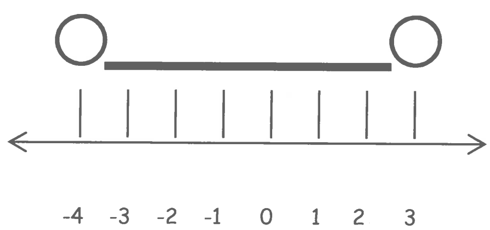
$R_5 =$ $[-3, \infty)$

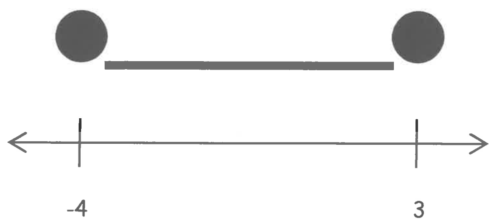
$D_6 =$ $(-\infty, -1]$

$R_6 =$ $(-\infty, 1]$

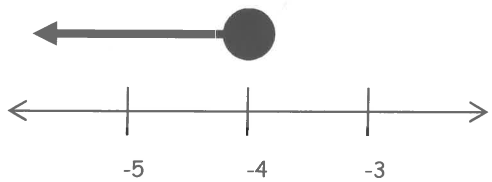
INTERVAL NOTATION

Interval notation is a way of expressing intervals other than using inequality notation (i.e. $\leq, >, \in R$, etc.)

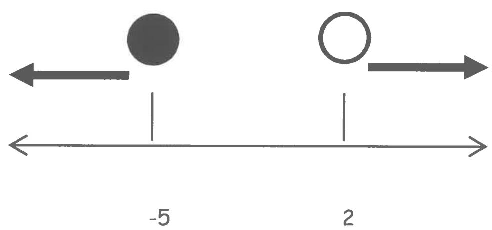
	Set Notation	Interval Notation
Ex.1 	$\{x \mid -4 < x < 3\}$	$(-4, 3)$ (open)

Ex. 2 

$\{x \mid -4 \le x \le 3\}$
$[-4, 3]$ (closed)

Ex. 3 

$\{x \mid x \le -4\}$
$(-\infty, -4]$ (half closed or half open)

Ex. 4 

$\{x \mid x \le -5 \text{ or } x > 2\}$
$(-\infty, -5] \cup [2, \infty)$
union joining of 2 sets

Do p. 293-296 #1 – 8, 10-12, 16
dom range

Ex. 5 Use interval notation to state the domain and range of:

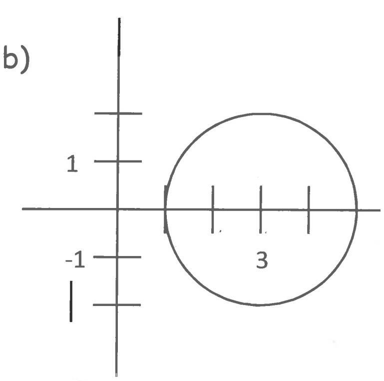
a) $y = 2x + 3$

any oblique line

D: $\{x \mid x \in \mathbb{R}\}$ $(-\infty, \infty)$

R: $\{y \mid y \in \mathbb{R}\}$ $(-\infty, \infty)$

b)



D: $\{x \mid 1 \le x \le 5\}$
$[1, 5]$

R: $\{y \mid -2 \le y \le 2\}$
$[-2, 2]$

5.6 **Properties of Linear Relations p. 300**

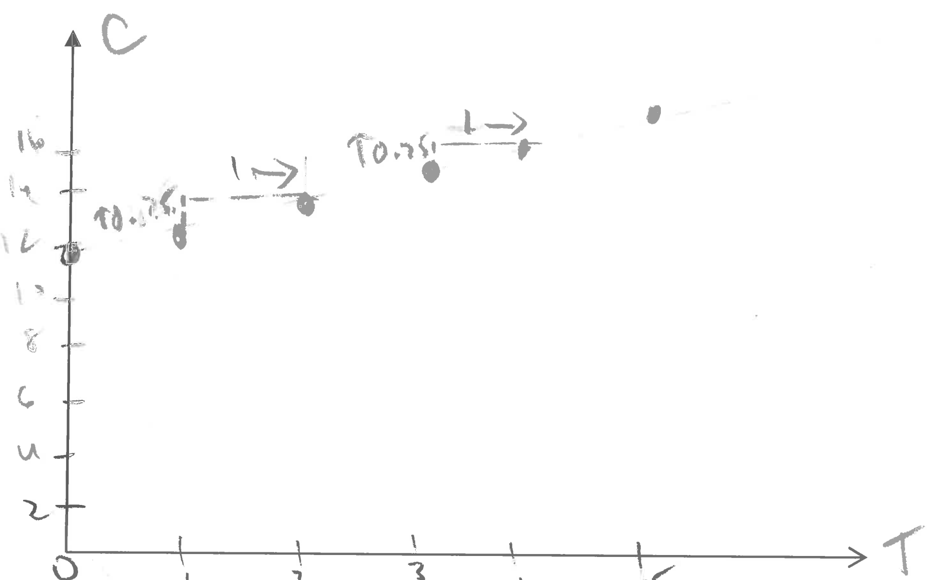
Learning Outcome: Learn to identify and represent linear relations in different ways.

intro 1 The table of values show the cost of a pizza with up to 5 extra toppings. p. 300

Number of Extra Toppings, T	Cost ($),c
0	12.00
1	12.75
2	13.50
3	14.25
4	15.00
5	15.75

0.75
0.75

Graph the data below and be sure to label the axes with the variables:(put the independent variable on the x-axis and dependent variable on the y-axis)



1. What patterns do you see in the table?

everytime: T goes up by 1; C goes up by 0.75

2. Write a rule for the pattern that relates the cost of a pizza to the number of its toppings.

$C = 0.75t + 12$ Cost is 0.75 times number of toppings plus 12.

3. How are the patterns in the table shown in the graph?

Dots are in a straight line.
Each dot is up 0.75 and over 1 from previous dot.
Each dot increases by same amt.

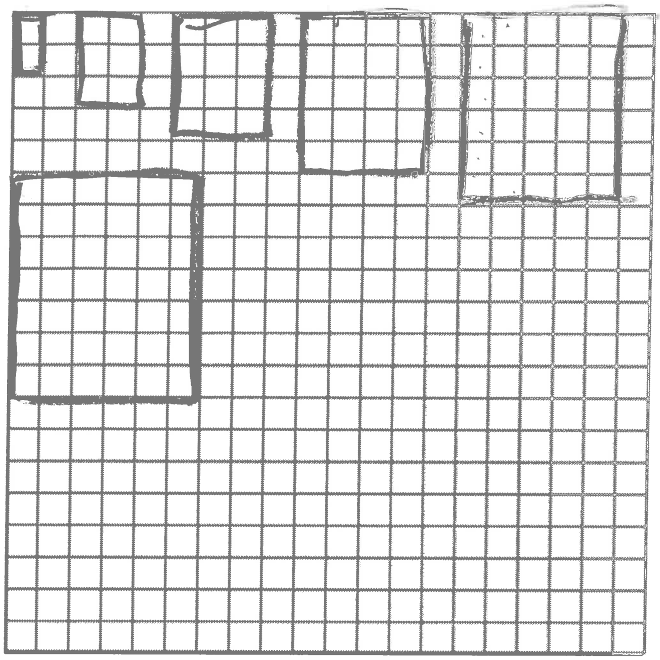
4. How can you tell from the table that the graph represents a linear relation?

Both sides have a constant difference.
Changes in T and C both constant.

40.

intro 2 Use the pattern of rectangles on pg. 301 in textbook. This pattern continues.

a) Draw the next two rectangles in the pattern. Copy and complete each table of values for the 6 rectangles.



Width of Rectangle, w (cm)	Area, a (cm²)
1	+4 2 1x2
+1 2	+6 6 2x3
+1 3	+8 12 3x4
4	+10 20 4x5
5	+12 30 5x6
6	42 6x7

Width of Rectangle, w (cm)	Perimeter, p (cm)
1	6 (1+2)2
2	10 (2+3)2 +4
3	14 (3+4)2 +4
4	18 (4+5)2
5	22 (5+6)2
6	26 (6+7)2

b. Which table of values represents a linear relation? relation between width + perimeter

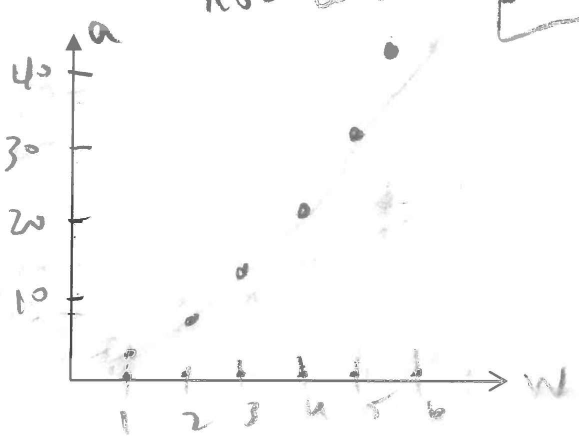
How can you tell?

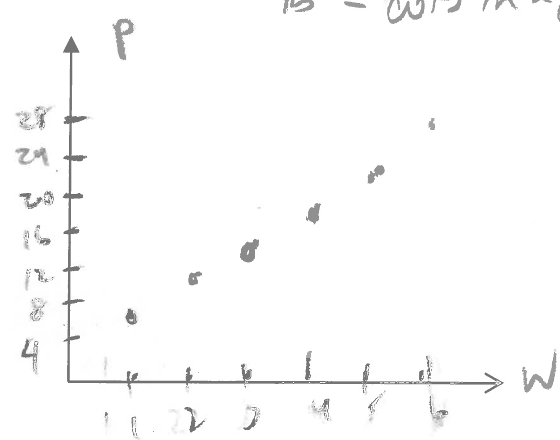
Constant difference of both variable. Width always +1. Perimeter always +4

c. Graph the data in each table of values. (Label each axis with a letter.) Does each graph represent a linear relation? How ~~do you know~~?

no - dots in a curve

Yes - dots in a line





We can identify that a relation is a linear relation in different ways. We can see these patterns using **table of values, ordered pairs or graphs**.

5.6 – PROPERTIES OF LINEAR RELATIONS

Consider renting a car for $60.00, plus $20.00 for every 100 km driven. The independent variable is the distance (km) and the dependent variable is the Cost.

There are different ways to show a linear relation.

- table of values

independent variable → | Distance (km) | Cost ($) | ← dependent variable

Distance (km)	Cost ($)
0	60
100	80
200	100
300	120
400	140

+100 +100 +100 +100

+20 +20 +20 +20

constant change in both variables

- ordered pairs

(independent, dependent)

+100 +100 +100 +100

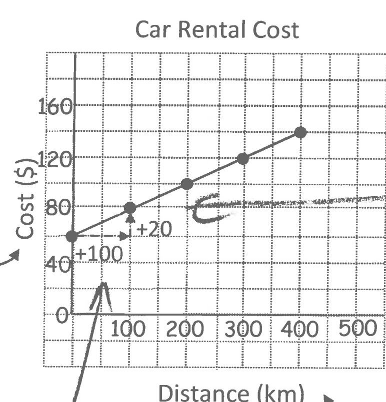
{(0, 60), (100, 80), (200, 100), (300, 120), (400, 140)}

+20 +20 +20 +20

- line graph

straight line -or dots that are in a straight line

dependent variable



change in dependent variable (vertical) +20

independent variable

change in independent variable (horizontal) +100

100km additional $20

We can determine the rate of change from the equation that represents the linear function.

The graph of a linear relation is a straight line. For a linear relation, a constant change in the indepenent variable results in a constant change in the dependent variable (rate of change).

(example p. 47 continued)

Rate of change

$$\frac{\text{Change in dependent variable}}{\text{Change in independent variable}} = \frac{\$20}{100\text{km}} = \$0.20/\text{km}$$

We can use each previous representation to calculate the rate of change. The rate of change can be expressed as a fraction:

The rate of change of $0.20/km means: for each additional km driven, the rental cost increases by $0.20 (20¢)

The rate of change is Constant for a linear relation.

The rate of change can also be determined from the equation that represents the linear function.

Let the cost be C dollars and the distance driven be d kilometres.

An equation for this linear function is: $C = 0.20d + 60$

initial amount

independant variable

rate of change

dependant variable

(taped out p. 42,43))

Intro 3 The cost for a car rental is $60, plus $20 for every 100km driven. The **independent variable** is the distance driven and the **dependent variable** is the cost. (*The cost DEPENDS on the distance driven.)*

1. **Table of Values**

Independent variable → | Distance (km) | Cost ($) | ← Dependent variable

	Distance (km)	Cost ($)	
	0	60	
+100	100	80	+20
+100	200	100	+20
+100	300	120	+20
+100	400	140	+20

You can see the distance and cost increase CONSTANTLY by the same number.

For a linear relation, a constant change in the independent variable results in a constant change in the dependent variable.

2. **Ordered pairs** –Ordered pairs are always written in same order: (*independent variable, dependent variable*). When writing a set of ordered pairs, it is important to write them with first elements (independent variables) in numerical order.

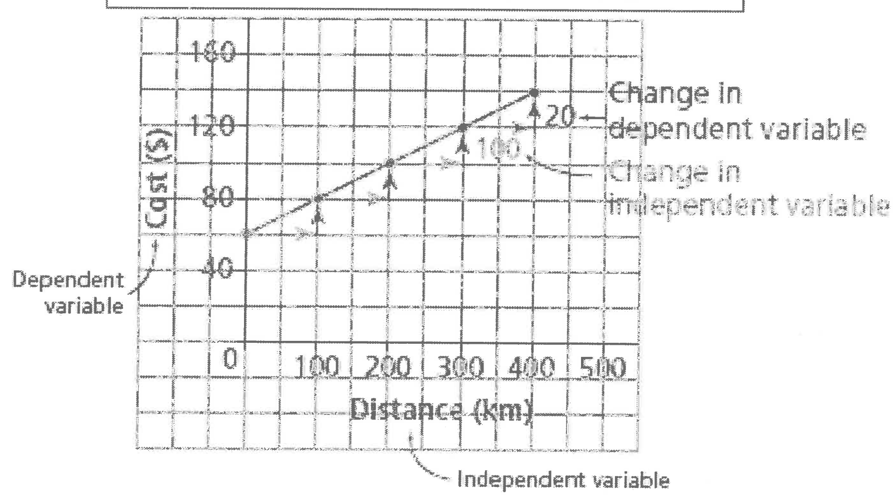
+100 +100 +100 +100

{(0, 60), (100, 80), (200, 100), (300, 120), (400, 140)}

+20 +20 +20 +20

3 **A graph**. - You can see that the **graph of a linear relation is a straight line.**

(Or if the graph is shown with discrete data points , the points go in the form of a line).



We can use each representation to calculate the **rate of change**. The rate of change can be expressed as a fraction. The dependent variable goes up by $20 each time. The independent variable goes up by 100 km each time.

Rate of change: $\frac{cha \quad in\ dependent\ variable}{chang \quad in\ independen \quad variable} = \frac{\$20}{100km} = \$0.20/km$

The rate of change is $0.20/km. For each additional 1km driven, the rental cost increases by 20 cents. **The rate of change is constant for a linear relation.**

Ex. 1 Look at example 1 a,b p. 303. Then try the following:

Consider each relation. Determine whether the relation is linear. Explain why or not.

1. Which table of values represents a linear relation? Justify your answer.

a) The relation between the number of bacteria in a culture, n, and time, t minutes.

t	n
0	1
20	2
40	4
60	8
80	16
100	32

20, 20, 20, 20

1, 2, 4, 8

not linear: changes isn't constant; changes in n are a pattern but not constant difference

b) The relation between the amount of goods and services tax charged, T dollars, and the amount of the purchase, A dollars

A	T
60	3
120	6
180	9
240	12
300	15

60, 60, 60

3, 3, 3

linear

changes in A and T are constant

Example 2 Look at example **2 a** and **b** p. 304 then try the following.

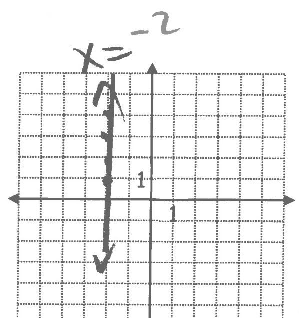
When an equation is written using the variables x and y,
x represents the *independent variable* and **y** represents the *dependent variable*.

2.a) **Create a table of values for each relation, and then graph the linear relation (using a ruler).** (*To graph the relation, draw intersecting axes using a ruler. Label with x and y and put arrows on axes. This is called a cartesian plane.*

Note: When an equation is written using the variables x and y, x represents the independent variable and y represents the dependent variable.

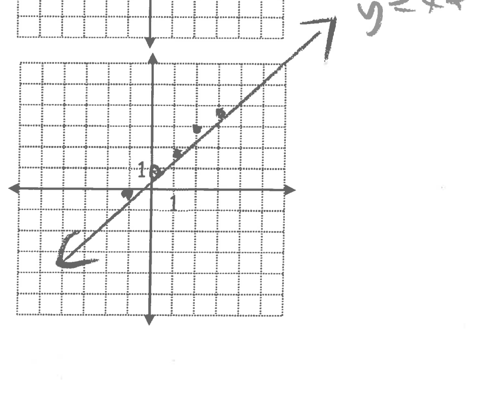
i) $x = -2$

x	y
-2	0
-2	1
-2	2
-2	3



ii) $y = x + 1$

x	y
-1	0
0	1
1	2
2	3
3	4

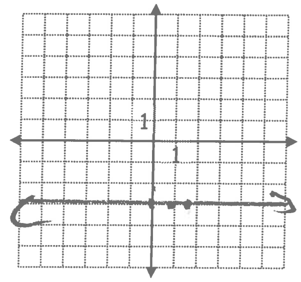


45

2 cont'd...

iii) $y=-3$

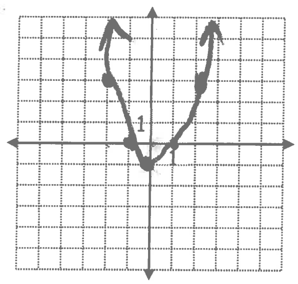
x	y
0	-3
1	-3
2	-3



$y=-3$

iv) $y=x^2-1$

x	y
-2	3
-1	0
0	-1
1	0
2	3



b) Which equations i - iv represent linear relations?

Graphs of i, ii, iii form lines, so they are linear relations.

Example 3 – Look at example **3 a** and **b** **p. 305** and then try the following.

3. Which relation is linear? Justify your answer.

a. A dogsled moves at an average speed of 10km/h along a frozen river. The distance travelled is related to time.

t(h)	d (km)
1	10
2	20
3	30

Linear - constant difference

constant change of 1

← constant change of 10

b. The area of a square is related to the side length of the square.

side length	area	
1	1	1^2
2	4	2^2
3	9	3^2

1, 1, 3, 5

not linear - area (second element) does not have constant difference

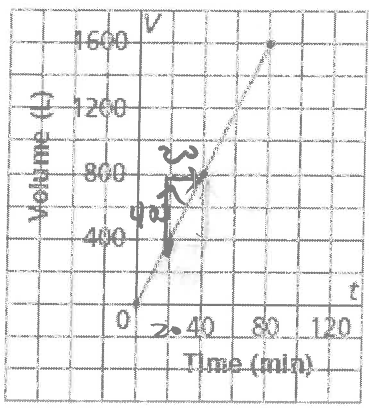
4b

Example 4 – Look at example **4 p. 306** -307 and then try the following.

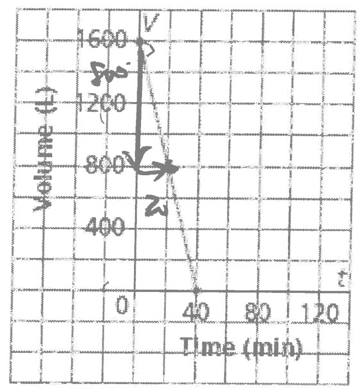
4. A hot tub contains 1600L of water. Graph A represents the hot tub being filled at a constant rate. Graph B represents the hot tub being emptied at a constant rate.

a. Identify the dependent and independent variables in each graph.

Graph A
Filling a Hot Tub



Graph B
Emptying a Hot Tub



Graph A

dependent variable: volume

independent variable: time

Graph B

dependent variable: volume

independent variable time

b) Determine the rate of change of each relation and describe what it represents.

Graph A ↑400 →20 ↑pos →pos

ROC = change in DV / change in IV

$= \frac{400}{20}$

= 20 L/min

ROC ↑pos ↑ so volume increases with time. ↗

Every minute 20 L added to tank.

Graph B ↓800 →20 ↓neg →pos

ROC = $\frac{-800}{20}$

= −40 L/min

Roc ↓neg so volume decreases with time ↘

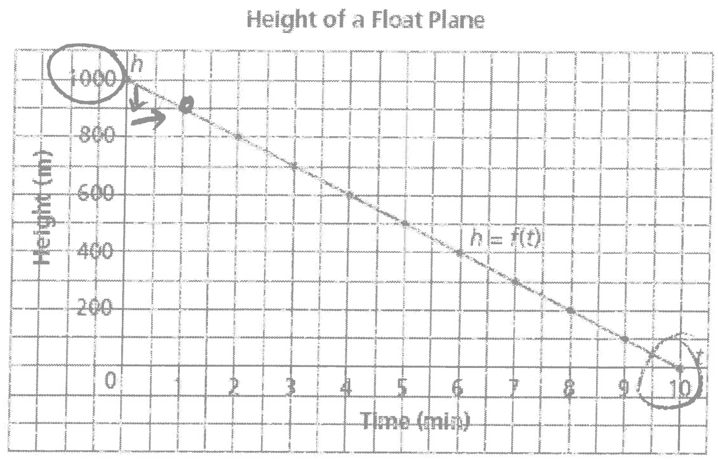
Every min 40 L drained from tank.

Do p. 307 – 308 #1 – 6, 7a, 10, 12

5.7 Interpreting Graphs of Linear Functions p. 311

Introduction Activity:

Float planes fly into remote lakes in Canada's Northen wilderness areas for ecotourism. This graph shows the height of a float plane above a lake as the plane descends to land.



1. Where does the graph intersect the vertical axis (y-axis)? 1000m

2. What does this point represent?

The height plane was at before descent

3. Where does the graph intersect the horizontal axis (x-axis)? What does this point represent?

10 min
amount of time it took for descent until landing

4. What is the rate of change for this graph? What does it represent?

Every minute the plane descends 100m.

5. State the domain, range and ~~rate of change~~ of this function.

RATE OF CHANGE: $\frac{900-1000}{1-0} = \frac{-100}{1} = -100$ m/min

↑ rate of descent
RoC negative

	SET NOTATION	INTERVAL NOTATION
DOMAIN	$\{0 \leq t \leq 10\}$	$[0, 10]$
RANGE	$\{0 \leq h \leq 100\}$	$[0, 100]$

~~Are there any restrictions on the domain/range? Explain.~~

Copy the definition for **Linear functions** (bottom p. 312)

Any graph of a line that is not vertical represents a function — a linear function

Read p. 313 then write definitions for ***Horizontal and Vertical Intercepts.*** (see also p. 315)

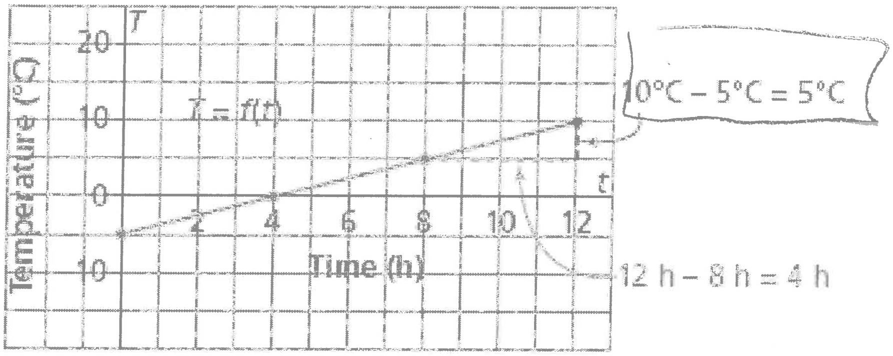
Horizontal Intercept (x-intercept): X-coordinate of point where graph intercepts x-axis

Vertical Intercept (y-intercept): y-coordinate of point where graph intersect y-axis

Intro 1 p. 313:

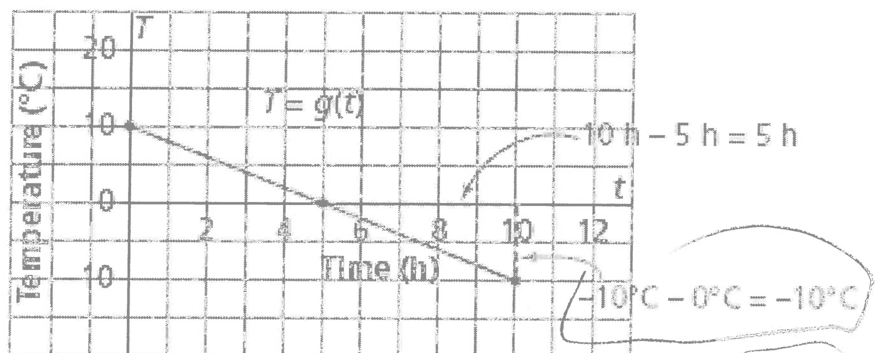
Each graph below shows the temperature of T degrees Celsius, as a function of, t hours, for two locations. Determine the horizontal and vertical intercepts, the domain and range, and the rate of change as well as describe what each represent.

Temperature in Location A



ascending
+ RoC
temp ↑ 5°c

Temperature in Location B



decreasing
RoC
neg
−10°c
temp ↓

Horizontal Intercept: 4

Coordinate (4, 0)

Vertical Intercept:

Coord (0, −5)

Domain: $\{0 \le t \le 12\}$

$[0, 12]$

Range: $\{-5 \le T \le 5\}$

$[-5, 5]$

Rate of Change:

$\frac{10-5}{12-8} = \frac{5}{4} = 1.25$ °c/h

RoC is pos (pos/neg) because the temperature is incr (incr/decr) over time.

(rises left to right)

Horizontal Intercept : 5

Vertical Intercept: 10

Domain: $\{0 \le t \le 10\}$

$[0, 10]$

Range: $\{-10 \le T \le 10\}$

$[-10, 10]$

Rate of Change:

$\frac{-10-0}{10-5} = \frac{-10}{5} = -2$ °c/h

RoC is neg (pos/neg) because the temperature is decr (incr/decr) over time.

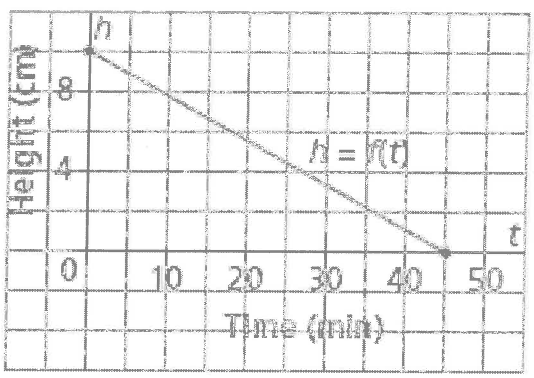
(falls left to right)

50

Example 1: Look at **example 1 p. 314** then try the following:

1. This graph shows how the height of a burning candle changes with time.

Height of a Burning Candle



a) Write the coordinates of the points where the graph intersects the axes. Determine the vertical and horizontal intercepts. Describe what the points of intersection represent.

b) What are the domain and range of this function?

a) (45,0) horiz 45
(0,10) vert 10

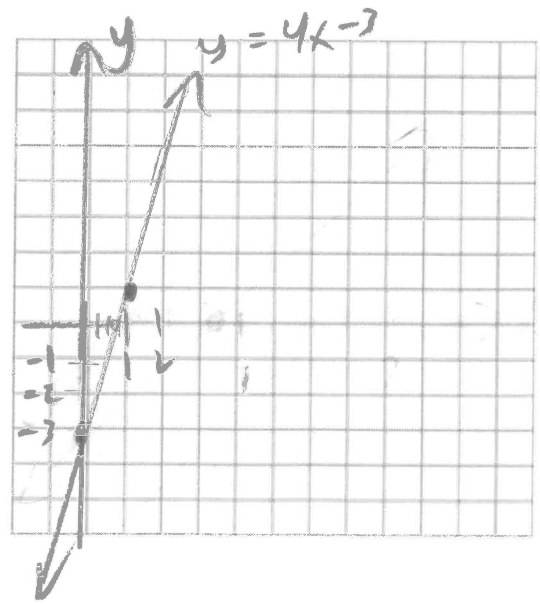
b) Dom $\{0 \le t \le 45\}$ $[0,45]$
Range $\{0 \le h \le 10\}$ $[0,10]$

Example 2: Look at **example 2 p. 315** then try the following:

- We can **use the intercepts** to **graph** a linear function written in function notation.
- **To determine the y-intercept** (vertical) evaluate f(x) when x=0, that is evaluate f(0) (sub in 0 for x solve for y)
- **To determine the x-intercept** (horizontal) determine the value of x when f(x)=0. (sub in 0 for y and solve for x)

2. Determine the x and y intercepts for the linear function f (x) = 4x – 3. Find a 3rd point on the graph. Use the intercepts and the 3rd point to graph the function. Draw the x and y axes with a ruler and label them with x and y.

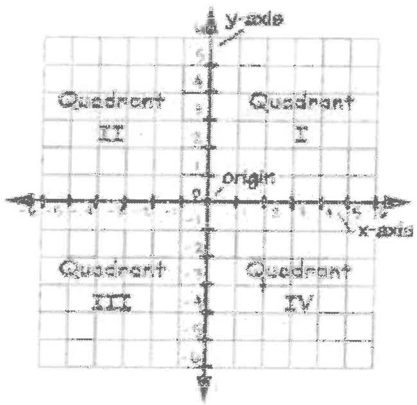
when x = 1
$f(1) = 4(1) - 3$
$= 4 - 3$
$= 1$
(1,1)

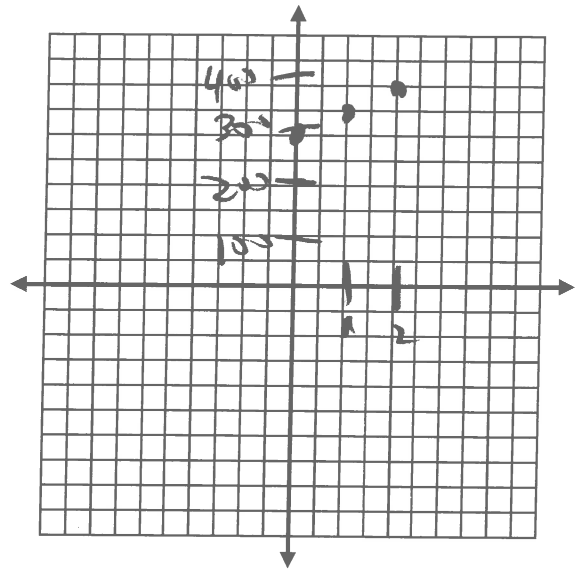


$f(x) = 4x - 3$
$= 4(0) - 3$
$f(x) = -3$
(0,−3)

$f(x) = 4x - 3$
$0 = 4x - 3$
$+3 \quad +3$
$\frac{3}{4} = \frac{4x}{4}$
$\frac{3}{4} = x$
$(\frac{3}{4}, 0)$

Interpreting Graphs of Linear Functions Assignment





Once every year Mrs Henry has to hire a pool company to fix open her pool for the summer. It costs $300.00 for the chemicals and the pool guy charges $50.00 per hour (if he works less than an hour, he rounds up to full hour). Create a table of values (define your variables). Write an equation to represent the cost of opening the pool for the year. Then graph it on the grid. Did you join the points? Why or why not?

$C = 50h + 300$

c = cost h = hours

No
- no partial hour cost
values between points have no meaning

h	c
0	300
1	350
2	400

Why does the graph not extend into quadrants II, III, or IV?

Can't have negative hours (II, III)

Can't have negative cost (IV)

b) What is the rate of change? (what number in the equation tells you rate of change and where can you find it on the graph?)

$ROC = \frac{350-300}{1-0} = 50$ choose 2 pts.

$\frac{\text{change in vertical}}{\text{change in horizontal}}$

coefficient of independent variable.

c) What is the fixed cost? (what number in the equation tells the fixed cost and where can you find it on the graph?) 300 - constant in equation beside the ROC + variable

y-intercept

52

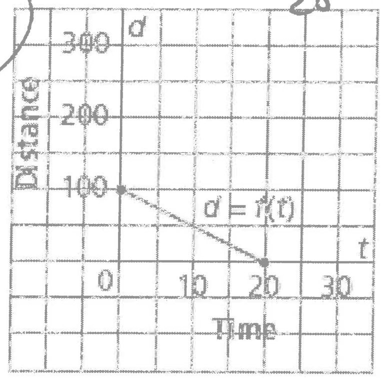
*Look at **example 3 p. 316** then try the following:*

3. Which graph has a rate of change of -5 and a vertical intercept of 100? Justify your answer.

ROC $\frac{0-100}{20-1} = \frac{-100}{20} = -5$

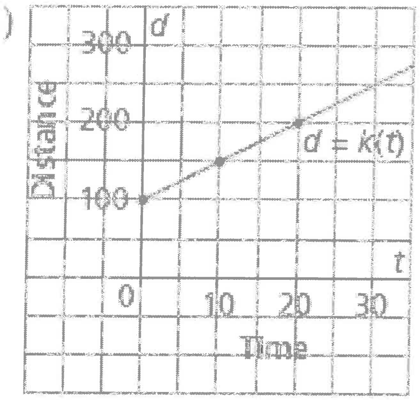
Increasing would have positive ROC

a)



decreasing neg. ROC

b)



Interpolation is the process of obtaining a value from a graph that is located between data points plotted.

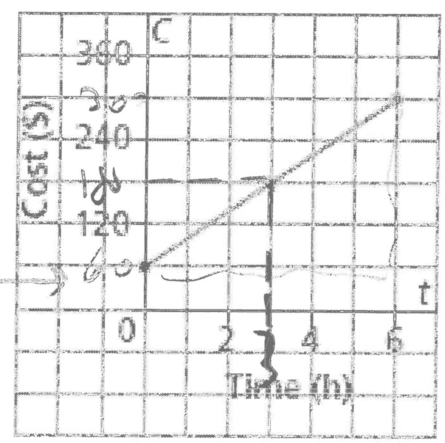
4. Look at **example 4 p. 317** then try the following.

4. This graph shows the total cost for a house call by an electrician for up to 6 h work.

The electrician charges \$190 to complete a job. For how many hours did she work?

no time = 60

Cost of an Electrician's House Call



Show your answers using both methods:
(a) interpolate by drawing horizontal/vertical dotted lines with ruler on graph ~3h
b) find answer by finding the rate of change and vertical intercept, then finding the equation, then substituting \$190 into equation for c to find t.

$\frac{300-60}{6-0} = \frac{240}{6} = 40$ \$/h.

$C = 40t + 60$
$190 = 40t + 60$
$-60 \quad -60$
$130 = 40t$

$\frac{130}{40} = \frac{40}{40}t$ 52
$\frac{13}{4} = t$
$3\frac{1}{4} = t$ $3\frac{1}{4}$ hours

The X- intercept is: x coordinate where the graph intercepts (crosses, touches) the x-axis

And can be found easily by:

a) substitute 0 for y (or for f(x)) in equation

b) solve for x

The Y – intercept is: y coordinate where graph intercepts y-axis

And can be found easily by:

a) substitute 0 for x in equation

b) simplify to find y

FINDING INTERCEPTS ALGEBRAICALLY

To find the y-intercept, substitute x = 0 into the equation, and solve for y.

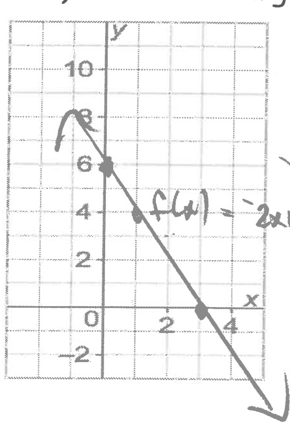
Evaluate f(x) when x = 0; that is, evaluate f (0).

To find the x-intercept, substitute y = 0 into the equation, and solve for x.

Determine the value of x when f(x) = 0.

SKETCHING A FUNCTION USING INTERCEPTS

a) Sketch a graph of the linear function f(x) = - 2x + 6



$0 = -2x + 6$
$-6 \qquad -6$
$\frac{-6}{-2} = \frac{-2x}{-2}$
$3 = x$

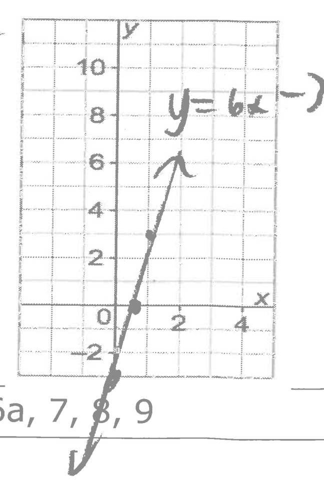
$f(x) = -2(0) + 6$
$= 6$

1 more point
$f(x) = -2(1) + 6$
$= 4 \quad (1, 4)$

$f(x) = 6(0) - 3$
$f(x) = -3$

$0 = 6x - 3$
$+3 \qquad +3$
$\frac{3}{6} = \frac{6x}{6}$
$\frac{1}{2} = x$

$6(1) - 3$
$= 3$
$(1, 3)$



b) Sketch a graph of the linear function f(x) = 6x – 3.

On pages 319-320 do the following questions: #1-3, 4, 6a, 7, 8, 9